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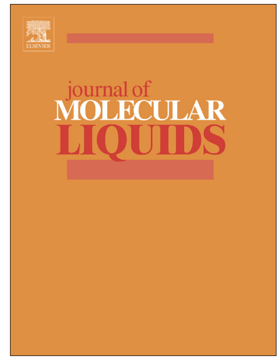

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Investigation of Lorentz forces and radiation impacts on nanofluid treatment in a porous semi annulus via Darcy law

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Abstract

Influence of Lorentz forces and radiative term on nanoparticles treatment through a porous semi annulus has been displayed in current article. An innovative numerical method was employed to portray the roles of radiative term, Rayleigh number and Lorentz forces. Working fluid is Al_2O_3-H_2O nanofluid with various shapes. Results display that inner wall temperature reduces with rise of Rd . The best shape of nanoparticle in view of hydrothermal treatment is Platelet shape.

***Keywords*:** Convective heat transfer; Radiation; Nanofluid; Darcy law; CVFEM; Lorentz forces.

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Nomenclature

Ra	Rayleigh number for porous media	θ	dimensionless temperature
Rd	Radiation parameter	β	Thermal expansion coefficient
Ha	Hartmann number for porous media	μ	Dynamic viscosity
T	Fluid temperature	Ω,ψ	dimensionless vorticity & stream function
k	thermal conductivity	ρ	Fluid density
C_p	specific heat capacity	β_R	Radiation coefficient
Nu	Nusselt number	σ	Electrical conductivity
MHD	magnetohydrodynamic	***Subscripts***	
B	Magnetic field	*nf*	Nanofluid
CVFEM	Control volume based finite element method	*loc*	Local
V	Vertical velocity	*ave*	average
Greek symbols			
σ_e	Stefan Boltzmann coefficient		

1. Introduction

Due to low thermal characteristics of common fluids, several ways have been offered to solve this issue. One of them is Nanotechnology. Dispersing metallic nanoparticles in to base fluid has been considered by various researchers. Bahiraei and Hangi [1] illustrated various applications of ferrofluid. They considered impact of magnetic source on treatment of ferrofluid. Mehmood et al. [2] demonstrated radiative Casson flow along a stretching plate. They demonstrated that friction factor augments with augment of mixed convection factor. Bahiraei et al. [3] displayed second law analysis for nanoparticle mixed convection in an annulus. They indicated that irreversibility enhances with augment of Richardson number. Impact of radiation on solidification mechanism was displayed by Sheikholeslami et al. [4]. They utilized CuO nanoparticles to accelerate this phenomenon. Soomro et al. [5] illustrated the nonlinear radiation impacts on nanofluid flow over a moving wall. They displayed that Nusselt number detracts with rise of thermophoresis forces. Hussain et al. [6] presented the mixed convection over a plate with considering radiation impacts. Their outputs revealed that convective flow rises with considering radiation. Rashid et al. [7] demonstrated inclined magnetic force influences on nanoparticle migration over a plate. They included thermal radiation in their simulation.

Sheikholeslami [8] portrayed the influence of magnetic force on phase change process. They employed adaptive grid in simulation. Atlas et al. [9] investigated radiation influence on squeezing flow of nanoparticles. They also considered the viscous dissipation effects. Nadeem et al. [10] demonstrated Lorentz forces effect on Casson fluid treatment over a plate. They solved the problem in three dimensional forms. Besthapu et al. [11] investigated slip flow effect on nanofluid behavior with radiative mode. Ahmed et al. [12] demonstrated Carbon nanotube transportation in a duct under the effect of radiation factor. Ramzan et al. [13] illustrated the nanofluid migration over a permeable plate. They portrayed that concentration field detracts with enhance of Brownian motion. Irfan et al. [14] portrayed the unsteady ferrofluid flow due to nonlinear radiation and chemical reaction. They utilized both semi analytical and numerical methods to simulate their three dimensional problem. Nanofluid has attracted many attentions in previous years [15-47].

In above articles, authors did not add the impact of nanoparticles shape in porous media. In this research, Lorentz forces effects on nanofluid transportation is simulated utilizing Darcy law. Innovative numerical approach (CVFEM) is used to model current problem. Shape of nanoparticles, radiative factor, Lorentz and buoyancy forces have been considered as active parameters of current problem.

2. Problem description

In current paper, porous semi annulus is modeled considering alumina nanofluid. Boundary conditions and shape of element were illustrated in Fig. 1. Uniform Lorentz forces affect the domain. Inner wall becomes warm with constant heat flux.

3. Governing equations and simulation

3.1. Formulation

Transportation of alumina nanofluid in a porous semi annulus was modeled with considering Lorentz forces. Permeable media is considered with Darcy model. So, we have following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\mu_{nf}}{K}u=-\frac{\partial P}{\partial x}+\sigma_{nf}B_0^2\left[(\sin\gamma)v\,(\cos\gamma)-u\,(\sin\gamma)^2\right] \tag{2}$$

$$\frac{\mu_{nf}}{K}v = -\frac{\partial P}{\partial y} + \left(T - T_c\right) g\, \rho_{nf}\, \beta_{nf} + \sigma_{nf} B_0^2 \left(\cos\gamma\right)\left[\left(\sin\gamma\right)u - v\left(\cos\gamma\right)\right] \tag{3}$$

$$\frac{1}{\left(\rho C_p\right)_{nf}} \frac{\partial q_r}{\partial y} + \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{nf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)\frac{1}{\left(\rho C_p\right)_{nf}}, \tag{4}$$

$$,\left[T^4 \cong 4T_c^3 T - 3T_c^4, q_r = -\frac{4\sigma_e}{3\beta_R}\frac{\partial T^4}{\partial y}\right]$$

The related formulas for nanofluid are:

$$\frac{(\rho\beta)_{nf}}{(\rho\beta)_f} = \phi\left[-1 + (\rho\beta)_s / (\rho\beta)_f\right] + 1 \tag{5}$$

$$\left(\rho C_p\right)_{nf} = \left(\rho C_p\right)_s \phi + (1-\phi)\left(\rho C_p\right)_f \tag{6}$$

$$\rho_{nf} = \rho_f (1-\phi) + \rho_s \phi \tag{7}$$

$$\frac{\sigma_{nf}}{\sigma_f} - 1 = \frac{3\phi(\Lambda - 1)}{\phi(1-\Lambda) + (2+\Lambda)}, \frac{\sigma_s}{\sigma_f} = \Lambda \tag{8}$$

μ_{nf} is calculated as:

$$\begin{aligned}
&\mu_{nf} = \mu_{static} + \mu_{Brownian} = \mu_{static} + \frac{k_{Brownian}}{k_f} \times \frac{\mu_f}{\Pr_f} \\
&k_{Brownian} = 5\times 10^4 c_{p,f}\, \rho_f\, g'(d_p, \phi, T)\phi \sqrt{\frac{\kappa_b T}{\rho_p d_p}} \\
&g'\left(d_p, \phi, T\right) = Ln\left(T\right)\left(a_1 + a_3 Ln\left(\phi\right) + a_4 Ln\left(d_p\right) Ln\left(\phi\right) + a_2 Ln\left(d_p\right) + a_5 Ln\left(d_p\right)^2\right) \\
&+\left(a_6 + a_{10} Ln\left(d_p\right)^2 + a_8 Ln\left(\phi\right) + a_7 Ln\left(d_p\right) + a_9 Ln\left(d_p\right) Ln\left(\phi\right)\right)
\end{aligned} \tag{9}$$

k_{nf} is obtained with following formula:

$$\frac{k_{nf}}{k_f} = \frac{(k_p - k_f)m\phi + mk_f + k_p(\phi+1) + k_f(1-\phi)}{k_p + (m+1)k_f + (k_f - k_p)\phi} \tag{10}$$

Characteristics, various shape factors and required coefficient have been displayed in Tables 1, 2 and 3 [48-49].

Non-dimension variable are defined as:

$$\Psi = \psi / \alpha_{nf}, \theta = \frac{T - T_c}{\Delta T}, \Delta T = \frac{q'' L}{k_f}, (X, Y) = (x, y)/L \tag{11}$$

Thus, we can reach the final forms:

$$\frac{\partial^2\Psi}{\partial Y^2} + \frac{\partial^2\Psi}{\partial X^2} = -Ha\frac{A_6}{A_5}\left[2(\cos\gamma)\frac{\partial^2\Psi}{\partial X\,\partial Y}(\sin\gamma) + \frac{\partial^2\Psi}{\partial X^2}(\cos^2\gamma) + \frac{\partial^2\Psi}{\partial Y^2}(\sin^2\gamma)\right] - \frac{A_3 A_2}{A_4 A_5}\frac{\partial\theta}{\partial X}Ra \tag{12}$$

$$\frac{\partial^2\theta}{\partial Y^2}\left(\frac{4}{3}\left(\frac{k_{nf}}{k_f}\right)^{-1} Rd + 1\right) + \left(\frac{\partial^2\theta}{\partial X^2}\right) = -\frac{\partial\theta}{\partial Y}\frac{\partial\Psi}{\partial X} + \frac{\partial\theta}{\partial X}\frac{\partial\Psi}{\partial Y} \tag{13}$$

Constants parameters were defined as:

$$Ha = \frac{\sigma_f K B_0^2}{\mu_f}, Ra = \frac{g K (\rho\beta)_f L \Delta T}{\mu_f \alpha_f}, Rd = 4\sigma_e T_c^3 / (\beta_R k_f)$$
$$A_4 = \frac{k_{nf}}{k_f}, \; A_5 = \frac{\mu_{nf}}{\mu_f}, A_3 = \frac{(\rho\beta)_{nf}}{(\rho\beta)_f},$$
$$A_2 = \frac{(\rho C_P)_{nf}}{(\rho C_P)_f}, \; A_6 = \frac{\sigma_{nf}}{\sigma_f}, \; A_1 = \frac{\rho_{nf}}{\rho_f}, \tag{14}$$

Furthermore, boundary conditions are:

$\theta = 0.0$ on outer wall (15)

$\Psi = 0.0$ on all walls

$\frac{\partial\theta}{\partial n} = 1.0$ on inner wall

Nu_{loc} and Nu_{ave} have been obtained as:

$$Nu_{loc} = \frac{1}{\theta}\left(1 + \frac{4}{3}\left(\frac{k_{nf}}{k_f}\right)^{-1} Rd\right)\left(\frac{k_{nf}}{k_f}\right) \quad (16)$$

$$Nu_{ave} = \frac{1}{S}\int_0^s Nu_{loc}\, ds \quad (17)$$

3.2. Numerical technique

CVFEM algorithm has been introduction as combination of FVM and FEM [50]. This approach has been used benefits of both previous approaches. Triangular element with linear interpolation was employed to estimate temperature and etc. The last step has been done via Gauss-Seidel method. Useful notes about this method have been explained in new reference book [50].

4. Grid analysis and validation

Several grid sizes should be tested to find the best one in which output does not rely on its size. As portrayed in table4, for this special case, size of 81×241 can be selected. Fig. 2 and table 5 display the validation of written code [51-53]. Good accuracy has been displayed for written code.

5. Results and discussion

In present article, nanofluid with various shapes has been used to portray the influences of shape factor on nanoparticle migration through a permeable media. Impact of radiative parameter, Brownian motion and magnetic forces have been taken into account. Graphs and tables portray the impact of Alumina volume fraction ($\phi = 0$ to 0.04), radiative term ($Rd = 0$ to 0.8), magnetic forces ($Ha = 0$ to 20) and Rayleigh number for porous media ($Ra = 100, 200$ and 600).

Table 6 portrays variation of Nu_{ave} for various shapes of nanoparticles. Using Platelet shape leads to greatest thermal conductivity. Therefore, highest Nu_{ave} is reported for Platelet nanoparticles. So, Platelet shape will be employed for further outputs. Fig. 3 illustrates roles of using Aluminium oxide. Velocity of working fluid augments with increase of ϕ due to interaction of nanoparticles together. Temperature gradient and $|\psi_{max}|$ enhance by dispersing nanoparticles. This phenomenon is negligible for higher Hartmann number.

Fig. 4, 5 and 6 display the influences of important parameters on contour plots. When there is no magnetic force, there are two vortexes which the upper one is stronger. Adding magnetic force makes $|\psi_{max}|$ to reduce. As Lorentz forces enhances the strength of vortexes reduces and domination mode convert to conduction. Higher buoyancy forces make the velocity to enhance and vortexes become stronger. It means that convection mechanism improves with rise of Ra. Shapes of isotherms are more complicated for greater Ra while isotherms become parallel together in existence of high magnetic force. Existing thermal plume shows the domination of convection mode. Thermal plume disappears with enhance of Harman number.

Nu_{ave} versus *Rd*, *Ha* and *Ra* is portrayed in Fig. 7. Following formula can use for current research:

$$\begin{aligned} Nu_{ave} &= 2.88 + 0.83Rd + 0.22Ra - 0.26Ha \\ &+ 0.064Rd\,Ra - 0.071Rd\,Ha - 0.22RaHa \\ &- 2.44 \times 10^{-4} Ra^2 \end{aligned} \tag{18}$$

Temperature near the hot wall decreases with enhance of thermal radiation and buoyancy forces. Thus, augmenting *Ra* and *Rd* leads to enhance in Nu_{ave}. By dominating conduction mode due to applying magnetic forces, temperature enhances at inner surface and in turn Nu_{ave} detracts with augment of magnetic forces.

6. Conclusions

Alumina nanofluid migration through a permeable semi annulus due to Lorentz forces has been displayed in current paper. Darcy law is considered for porous terms. Radiation term, Brownian motion and shape factor influences are added in governing equations. CVFEM simulations portray the impacts of radiative factor, shape factor, buoyancy forces and Hartmann number. Outputs portray that Nu_{ave} augments with enhance of Ra, Rd . Vortexes become weak in presence of Lorentz forces.

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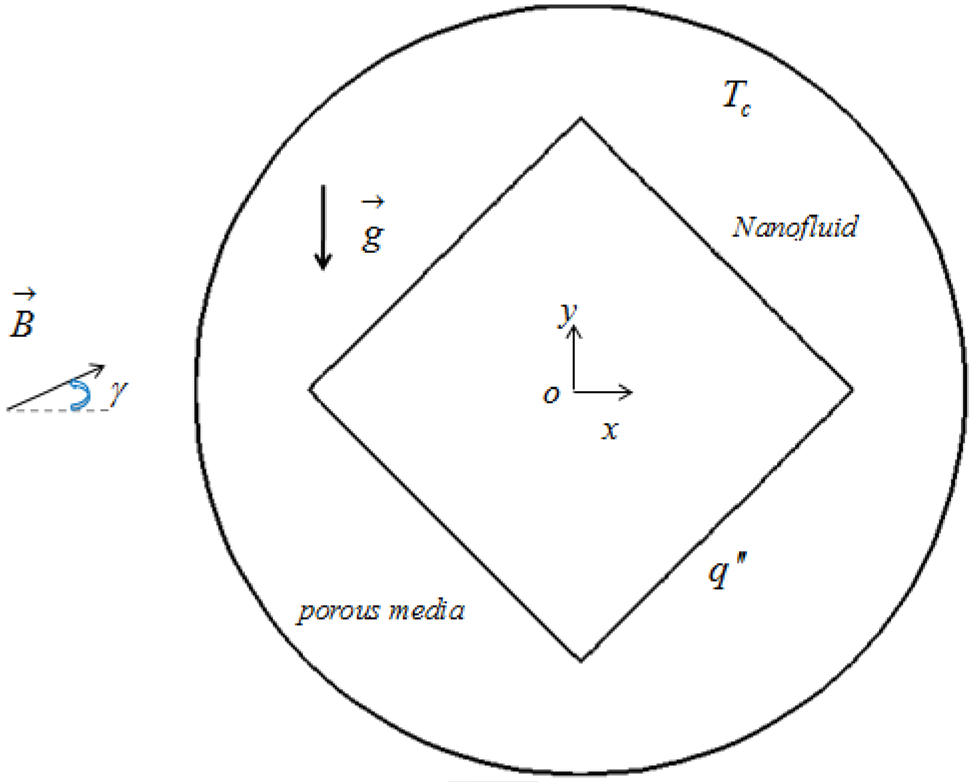
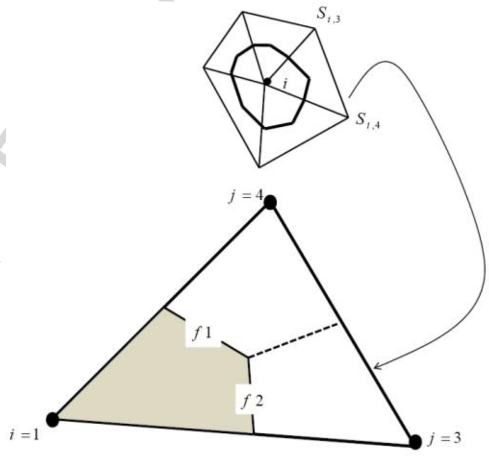


(a)

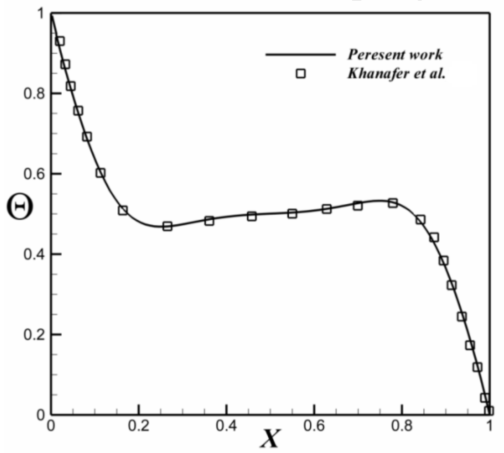


(b)

Fig. 1. Geometry and sample triangular element

Present result Kim et al.[22]

(a)



(b)

Fig. 2. Validation of current code with (1) Kim et al. [51] when Ra= 10^5, Pr=0.7; (b) Khanafer et al. [52] at $Gr = 10^4$, $\phi = 0.1$ and $Pr = 6.2(Cu - Water)$.

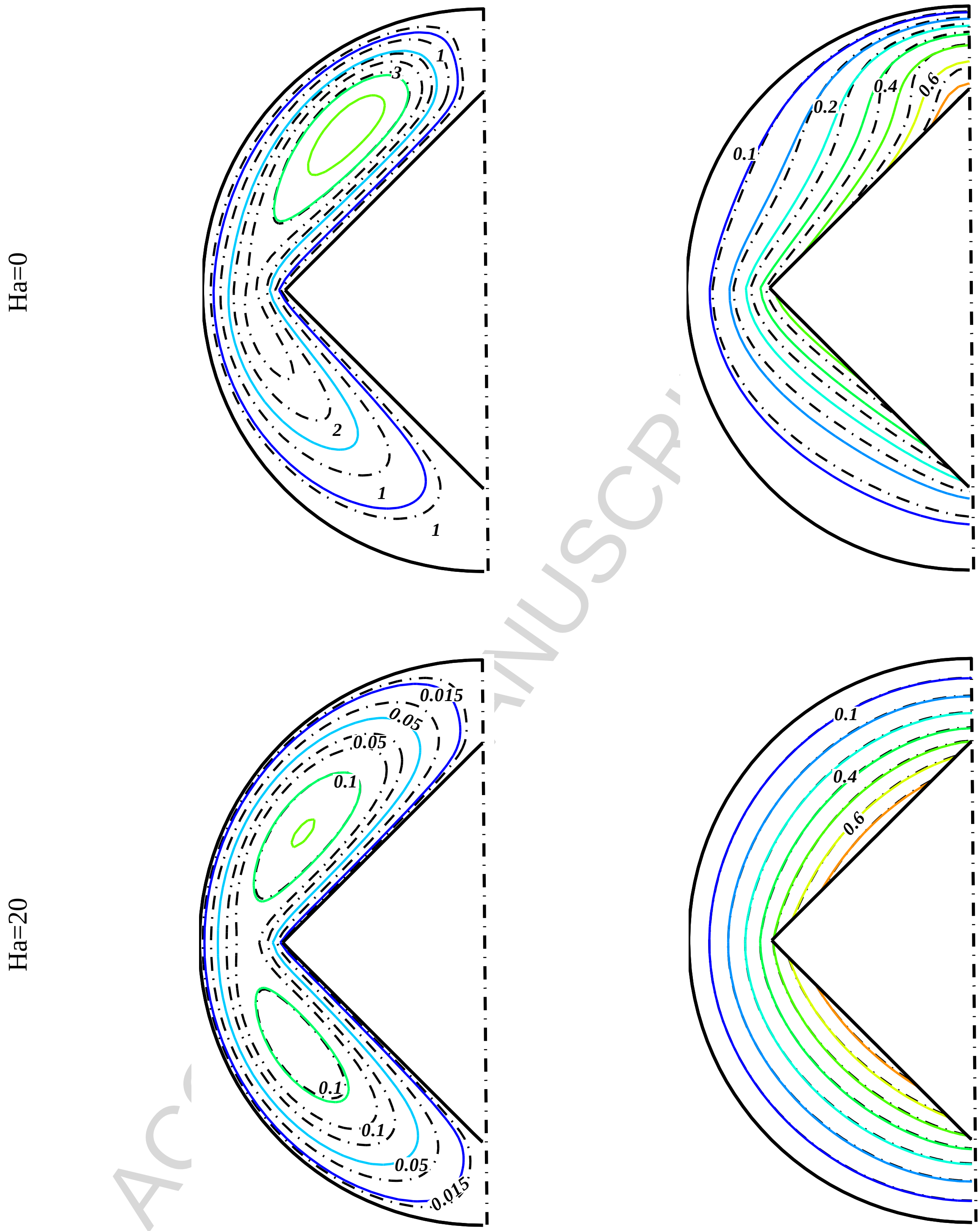


Fig. 3. Effect of nanofluid volume fraction on streamlines (left) and isotherms (right) contours (nanofluid ($\phi = 0.04$)(—) and pure fluid($\phi = 0$) (- - -)) when $Ra = 600$, $Rd = 0.8$

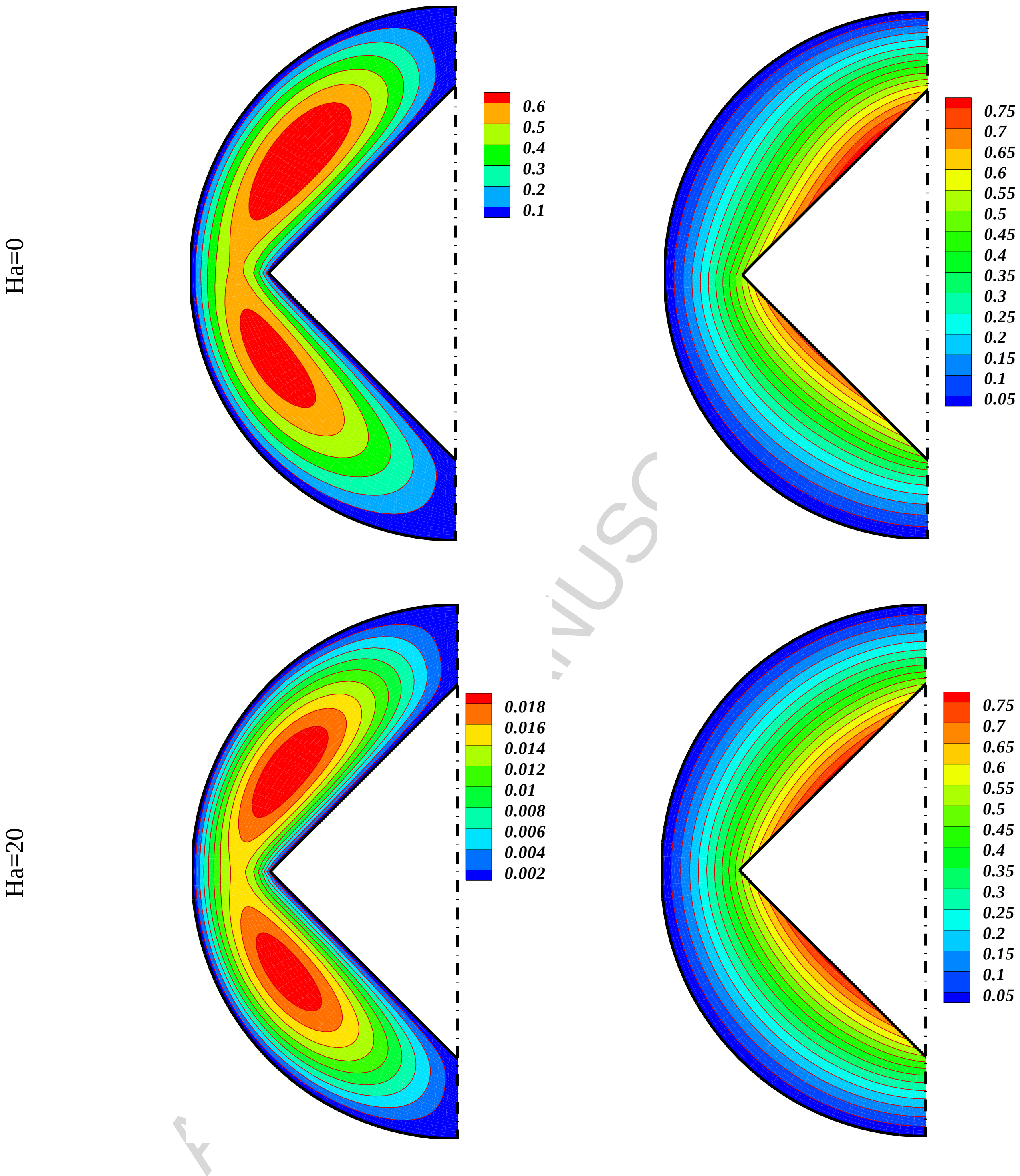


Fig. 4. Influence of Ha on streamlines (left) and isotherms (right) at $Ra = 100$, $Rd = 0.8$

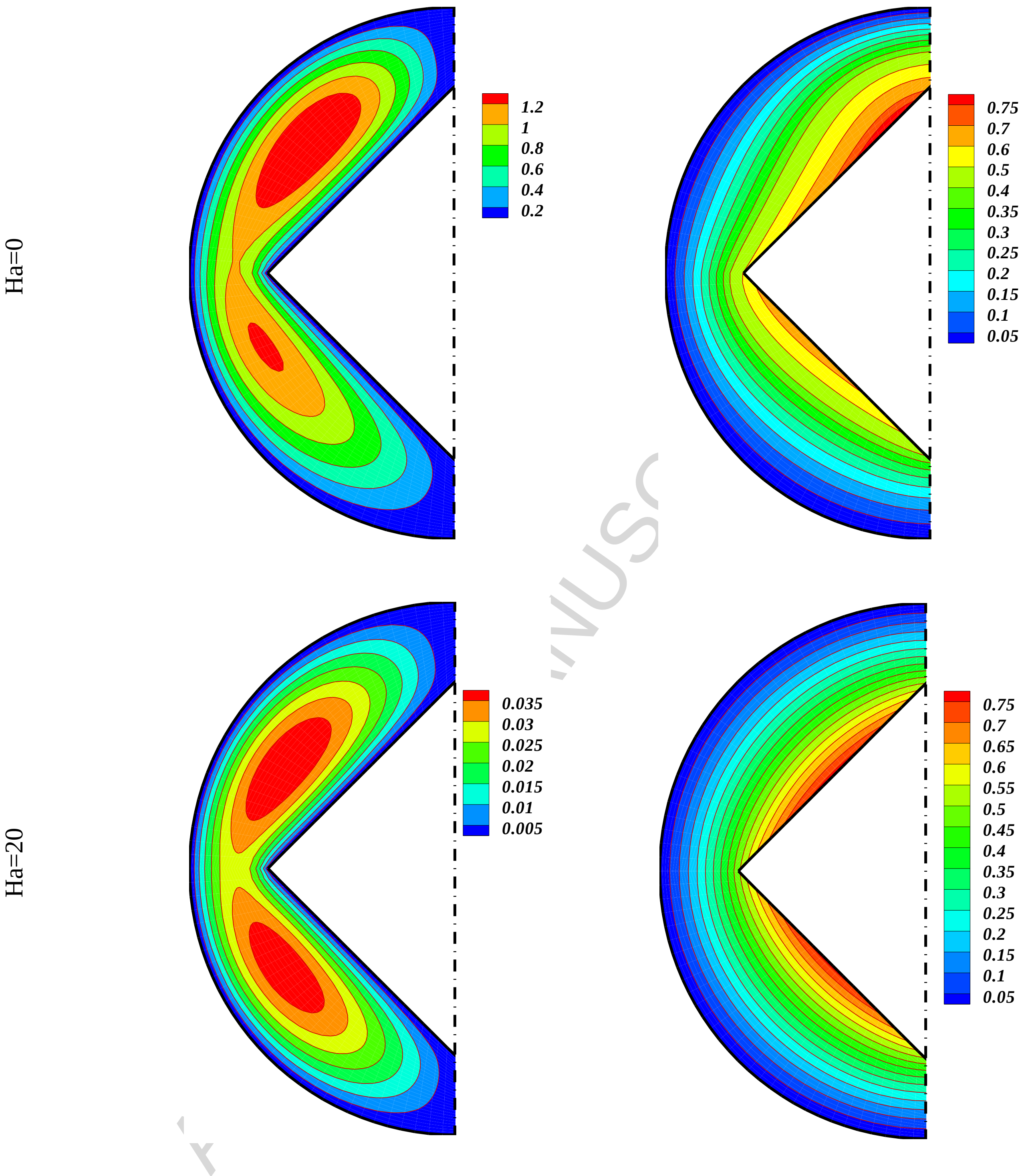


Fig. 5. Influence of Ha on streamlines (left) and isotherms (right) at $Ra = 200, Rd = 0.8$

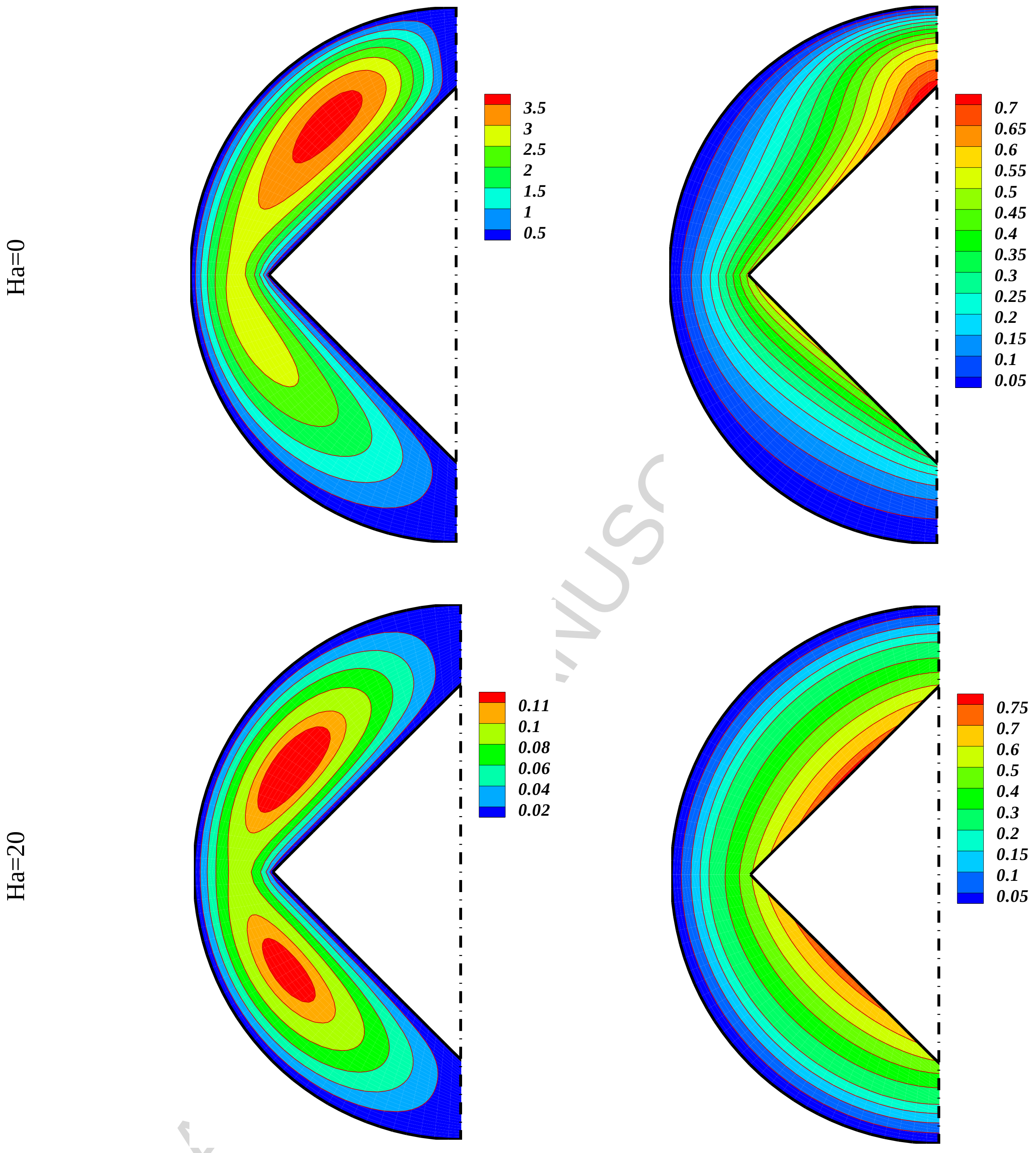
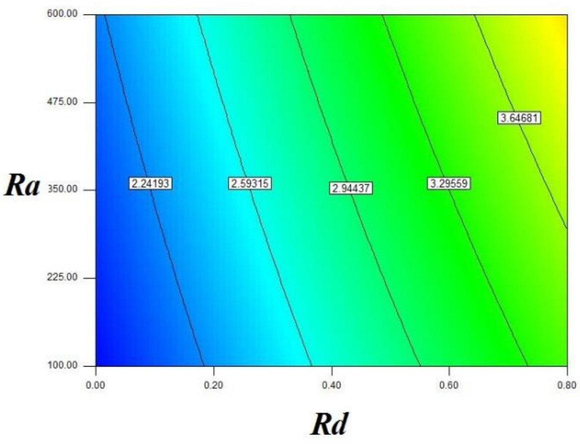
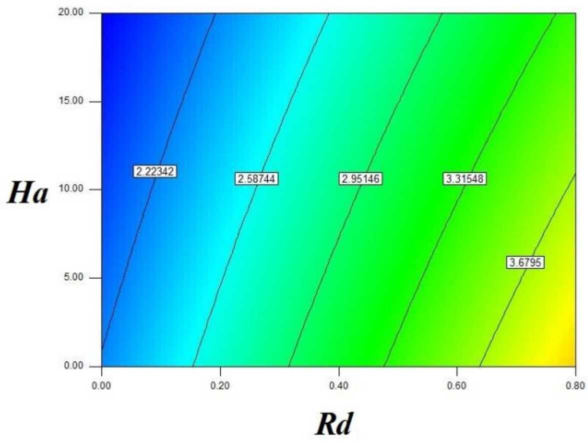


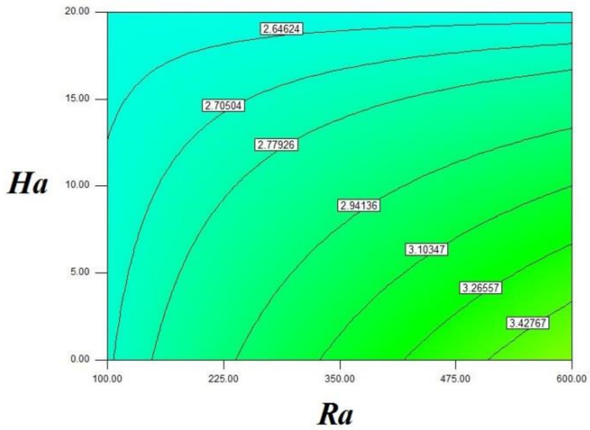
Fig. 6. Influence of Ha on streamlines (left) and isotherms (right) at $Ra = 600, Rd = 0.8$



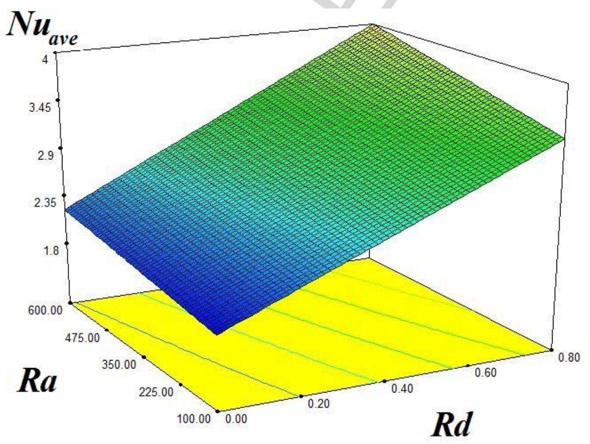
$Ha = 10, \phi = 0.04$

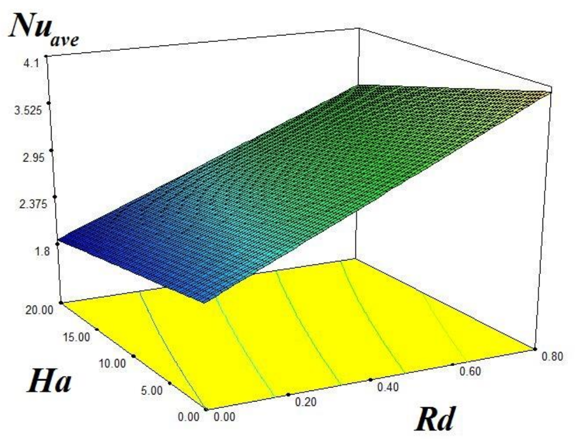


$Ra = 350, \phi = 0.04$

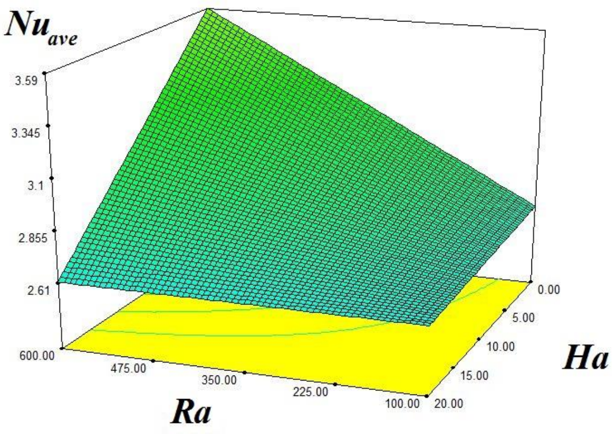


$Rd = 0.4, \phi = 0.04$





$Ha = 10, \phi = 0.04$ $Ra = 350, \phi = 0.04$



$Rd = 0.4, \phi = 0.04$

Fig. 7. Impacts of Ha, Ra and Rd on Nu_{ave} .

Table 1. Coefficient values of $Al_2O_3-Water$ [48]

Coefficient values	$Al_2O_3-Water$
a_1	*52.813488759*
a_2	*6.115637295*
a_3	*0.6955745084*
a_4	*4.17455552786E-02*
a_5	*0.176919300241*
a_6	*-298.19819084*
a_7	*-34.532716906*
a_8	*-3.9225289283*
a_9	*-0.2354329626*
a_{10}	*-0.999063481*

Table 2. Characteristics of water and nanoparticles [48]

	$\rho(kg/m^3)$	$C_p(j/kgk)$	$k(W/m.k)$	$\beta\times10^5(K^{-1})$	$\sigma(\Omega\cdot m)^{-1}$
Water	*997.1*	*4179*	*0.613*	21	*0.05*
Al_2O_3	*3970*	*765*	*25*	0.85×10^{-5}	1×10^{-10}

Table 3. Shape factor for various shapes of nanoparticles [49]

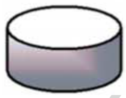
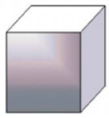
m	Spherical		3
	Platelet		5.7
	Cylinder		4.8
	Brick		3.7

Table 4. Mesh independency according to Nusselt number when $Ra = 600$,

$Rd = 0.8, Ha = 20$ and $\phi = 0.04$.

Mesh size in radial direction $\times$ *angular direction*				
51×151	61×181	71×211	81×241	91×271
3.348774	*3.35965*	*3.368493*	*3.369031*	*3.369755*

Table 5. Code validation for MHD flow at Pr=0.733.

Ha	$Gr = 2\times10^4$		$Gr = 2\times10^5$	
	Present	Rudraiah et al. [53]	Present	Rudraiah et al. [53]
0	2.5665	2.5188	5.093205	4.9198
10	2.26626	2.2234	4.9047	4.8053
50	1.09954	1.0856	2.67911	2.8442
100	1.02218	1.011	1.46048	1.4317

Table 6. Influence of shape factor on Nusselt number when $Rd = 0.8, Ra = 600, \phi = 0.04$

Shape	**Ha**	
	0	**20**
Spherical	4.516754	3.213638
Brick	4.545081	3.253901
Cylinder	4.59018	3.317008
Platelet	4.627558	3.368493

Highlights

- Water based nanofluid radiative heat transfer in a porous annulus is presented.
- New numerical approach (CVFEM) has been applied.
- Nu enhances with rise of the thermal radiation.
- Nu detracts with the augment of magnetic forces.