RELATION AMONG FLUID FLOW OVER A LINEAR, NONLINEAR AND EXPONENTIALLTY STRETCHING/SHRINKING SHEETS.

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Dedication

My devoted husband and respected teachers

I dedicate this to my devoted husband, a revered teacher whose prayers and encouragement have consistently served as an inspiration.

Acknowledgments

Praise be to Allah, who taught by pen. I am grateful to the Almighty who made me learn and achieve this milestone. Best prayers and peace be upon His best messenger Hazrat MUHAMMAD ﷺ who is the beacon of light for mankind.

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Abstract

The following steps in the current thesis establish the idea of the relation among fluid flow over linear, nonlinear and exponential stretching sheets. Following are the steps taken to construct the thesis framework. An extensive body of literature on the relation among fluid flow over linear, nonlinear, and exponential stretching sheets is studied in the first chapter. For linear, non-linear and exponential stretching/shrinking sheets, precise features of fluid flow, boundary layer theory, momentum equation, and heat transfer are explored. In addition to this, the second chapter explores basic fluid terminologies, characteristics and fundamental rules. The third chapter looks at a paper that investigates a linear/nonlinear stretching sheet of governing the fluid flow. One special situation that can be considered is a linear stretching sheet. The governing equation of momentum, continuity and energy are constructed by using tensor analysis. By making use of similarity variables, governing partial differential equations (PDEs) were transformed into dimensionless non-linear ordinary differential equations (ODEs) and solved numerically. In the next chapter, a numerical investigation of a fluid flow over an exponentially stretching sheet in the presences of thermal radiation is carried out. By using helpful similarity variables, governing PDEs were translated into dimensionless non-linear ODEs and then numerically solved. A comparative analysis of fluid flow for linear, nonlinear and exponentially stretching/shrinking sheet has been carried out in chapter five. Chapter six gives comprehensive detail of the major findings of this dissertation.

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NOMENCLATURE

$\bar{u}, \bar{v}$	Component of velocity
x	Coordinate along the plane
y	Coordinate along the normal
T	Fluid Temperature
T_w	Surface Temperature
U_∞	Velocity at plane
T_∞	Ambient temperature
μ	Dynamic viscosity
$C_{\boldsymbol{p}}$	Heat capacity
ρ	Density of the fluid
ν	Kinematic viscosity
K	Thermal conductivity of viscous fluid
C,L	Constant numbers
q_r	Radiative Heat flux(K/m)
E	Eckert number
$U_{\mathbf{0}}$	Source velocity
T_0	The plates' temperature
Re	Reynold number
T^4	Linear temperature function
σ^*	Constant of Stefan Boltzmann
k^*	Consumption co-efficient
K	Radiation number$\left(\frac{4\sigma^* T_\infty^3}{k^* k}\right)$
Nu_x	Local Nusselt number
$\bar{T}_w$	Skin friction against the wall
λ	Stretching parameter
C_f	Skin friction
q_w	Heat flux
λ	Stretching/shrinking parameter
Pr	Prandtl number

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

A thin layer of flowing viscous fluid near to the surface is known as the boundary layer. It has a wide range of uses, including movable lids, heat enrichment, sport aerodynamics, and aircraft. Prandtl [**1**] initially suggested the concept of boundary layers. He divided the flow field into two areas and discussed it in his research. The first area is a thin layer (boundary layer) near to the body where viscosity predominates. The second area is outside of this layer, and it is the location where a fluid particle's velocity reaches 0.99% of free stream velocity. With respect to distance from the surface velocity will fluctuate for a thin layer. A thin layer above the surface is known as the boundary layer and has a velocity gradient., Sakiadis [**2**] investigated boundary layer flow on a solid surface. He examined the boundary layer solution using both close and broad approaches. During 1970s, crane [**3**] investigated the boundary layer flow on a stretching/shrinking sheet. When the velocity varies, he established an exact solution for, 2-dimensional incompressible boundary layer flow.

The extrusion methods used in the industry of plastic and metal both utilize the flow generated by a stretching boundary [**4**]. A ground-breaking investigation was conducted by Sakiadis to examine the motion of boundary layer on a continually stretching moving surface at a steady speed. According to Lee [**5**] the flow through a needle with different diameters is where the study on boundary layer flows across a thin object with variable thickness was originally discovered in the past. The boundary layer flows through a thin needle were then extensively studied [**6-8**] while taking heat transfer effects into account.

These studies have an impact on the issue of polymer sheets that are continuously extruded from pigment. A common belief is that the sheet is nonprotractile, however as noted by Crane [**9**], circumstances may develop within the polymer sector once it is required to cope with a stretching plastic sheet. The movement of the boundary layer past a wall that is stretched at a velocity proportional to the distance with the wall was studied by Dangberg and Fenster [**10**] for a non-identical solution. Gupta and Gupta [**11**] studied the behavior of boundary layer over a stretching/shrinking sheet subject to suction for the velocity and temperature transport related to the given solution. The characteristics of heat transport over a continuously expanding surface with varying surface temperature were investigated by Chen and Char [**12**].

Rajagopal et al. [**13**] studied the viscoelastic fluid's flow behavior over a stretching sheet provided an approximation of the flow field's solution. The difficulty of Rajagopal et al. [**13**] was recently solved precisely by Troy et al. [**14**].

Afzal and Varshney [**15**], Kunken [**16**], and Banks [**17**] generated the sheet from a linearly stretching state to one that stretches with a power-law velocity. Mair Khan et .al [**18**] report the physical properties of a linearly stretched sheet and an MHD tangent hyperbolic fluid containing nanoparticles. In their investigation of the magnetohydrodynamic Eyring-Powell fluid flow over a stretched surface, Akbar et al. [**19**] used the shooting method to compute the numerical solution. They discovered that the velocity profile's Hartmann number is negative. Recently, many researchers discussed the impact of applied magnetic field on non-Newtonian fluid flows over stretched surfaces [**20-22**]

Vajravelu et al. [**23**] examines the stream of the viscous flow with nonlinear extended sheet, where it was determined the characteristics of transfer of heat. Later, they conclude this phenomenon in terms of numerical technique [**24**] which also includes straightly extending sheet issue. Cartel [**25**] explorer transfer of heat in thick liquid (viscous flow) where two cases have been discussed include sheet with consistent temperature and sheet with endorsed temperature. After this conclusion Cortoll [**26**] also examines flux in transfer of temperature in viscous flow in the presence of radiations. Ishak et al. [**27**] explorer the collision of extending sheet on MHD. Hammad et al. [**28**] explore the two-dimensional flow and heat transfer of an incompressible viscous nanofluid via a non-linear stretching sheet. The overall framework is about the viscous flow in presence of MHD boundary layer.

Fluid in which nano size particle with 1/100nm length are suspended is called nanofluid. Nadeem et al. [**29**] explore the effect of nanoparticles on the viscous flow over the extending sheets. Recently Reddy et al. [**30**] explorer the slushy dissemination effect on free convection MHD fluid overextending surface including chemical responses. To approve the comes about a cooperative consider between the display consider and already distributed comes about for a specific case is conducted and great understanding is found between them.

Javad et al. [**31**] presumes the turning stream of an incompressible laminar flow over the stretching plane including exponential sheet. Liu et al. [**32**] conclude the transfer of heat in 3-D laminar flow which examines the surface extended exponentially along horizontal direction. This simulation of reflection accept that the surface temperature spreads more exponentially and the change in similarity is used to reduce the monitoring conditions to a set of ordinary differential equations.

Hayat et el. [**33**] studied the three-dimensional flow effects on thermal diffusion and energy flux including heat source and reactions. Later on, Bhattacharyya et all. [**34,35**] explore the different manners of chemical reactions on MHD flow over the vertical stretching sheet. On the other hand, Skiadas [**36**] explore for the first time the different exploits of viscous fluid an account of moving plane. According to this definition, thermal radiation is the process through which a heated surface radiates energy in the form of electromagnetic radiation in all directions. Energy is transported across material by the process of radiation, either as waves or particles. Three types of radiation are distinguished: sound, energy, and light. Energy transfer in the synthesis of polymers and fossil fuels, as well as in astrophysical fluxes, is calculated using thermal radiations. Among other applications, thermal radiation is crucial for space exploration, high-temperature activities, and controlling the heating process in the polymer industry.

Elbashbeshy and Damian [**37**] examined boundary layer flow across a wedge with viscosity coefficient while considering the radiation effect and heat transfer. He used a fourth order Runge-Katta technique to resolve a problem involving the impact of radiation on Blasius flow. Later, by using the homotropy analysis method to solve the problem analytically, Sajid and Hayat [**38**] addressed the impact of thermal radiation on viscous flow produced by an exponential stretched sheet (HAM).

Our main objective was to create numerical solution using graphical comparisons of linear, nonlinear, and exponential stretching sheet. We were intended to show the velocity and temperature among the said stretching sheet and combined them as one. Chapter 1 of this research introduces the topic and discusses literature review. The next chapter enlightens about the basic definitions and concepts around which the study revolves. Chapter 3 encloses a detail study of boundary layer flow of a viscous fluid along linear and nonlinear stretching sheet. In chapter 4 a numerical investigation of flow of an exponential stretching sheet with thermal radiation is carried out. The flow problem is mathematically modeled in the form of nonlinear partial differential equations (PDEs) and converted into ordinary differential equations (ODEs) along with the boundary condition. Then the set of ODEs were solved numerically by using bvp4c MATLAB built-in command.

CHAPTER 2

BASIC DEFINITIONS

This chapter quickly discusses a number of fundamental terms, definitions and laws relating to the fluid movement and heat transfer.

2.1 Fluid

A substance that cannot withstand shear stress is said to be fluid. A fluid will constantly deform when shear stress is applied. The fluid's velocity or flow is interpreted as the deformation. Fluids, which include both liquid and gas, have different properties from solids. There are essentially two categories of fluids. Fluids that are Newtonian and non-Newtonian (both are actual fluid).

2.2 Fluid Mechanics

The properties of fluids in motion or at rest were of interest to this section of mechanics. It is divided into three sections. Kinematics, fluid dynamics, and static fluid

2.2.1 Kinematic fluid

The study of fluid particle motion in the absence of external forces is known as fluid kinematics. This also addresses the velocities and accelerations of moving fluid particles.

2.2.2 Dynamic fluid

Analysis of the movement of the particles contained in a fluid is known as fluid dynamics. It can be applied to examine the airflow over an aero plane wing or over the surface of a car. To boost the speed at which ships navigate the water. It can also be incorporated in their design.

2.2.3 Static fluid

A fluid layer cannot mov**e** in relation to an adjacent layer, and there are no shear forces in the fluid, according to the branch of fluid mechanics known as fluid static.

2.3 Classification of fluid:

2.3.1 Viscous fluid

Viscous fluids are defined as having a higher thickness or viscosity; they are typically very gloppy fluids. Shampoo and motor oil, as examples.

2.3.2 non-viscous fluid

A non-viscous fluid is one that has no internal friction or barrier to flow or viscosity.

Example: Superfluid liquid helium.

Figure 2.1: superfluid

(https://hicodenver.com/2017/12/strange-but-true-superfluid-helium-can-climb-walls/)

2.3.2.1 Superfluid

A fluid which is ability to flow indefinitely without losing any energy is called superfluid.

2.3.3 Real fluid

Real fluids have **s**ome viscosity and can be compressed.$(\mu \neq 0)$.Petrol and castor oil are examples of real fluid.

2.3.4 Ideal fluid

It is an incompressible, viscous fluid that cannot be compressed. Practically speaking, this kind of fluid cannot exist. ($\mu = 0$).

2.3.5 Compressible fluid

When a fluid's density directly correlates with its temperature and pressure, the term compressible fluid is used. Gases are one of the most typical examples.

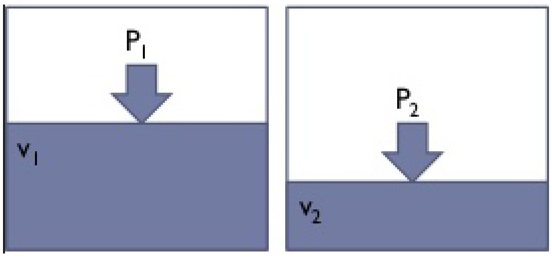

Figure 2.2: Compressible fluid

(https://m.facebook.com/prepareyourself.gk/photos/what-is-incompressible-fluida-fluid-in-which-the-density-remains-constant-for-is/106071131116820/)

2.3.6 Incompressible fluid

An incompressible fluid is one whose density is constant independent of the temperature or pressure. Liquids are typically thought of as being incompressible.

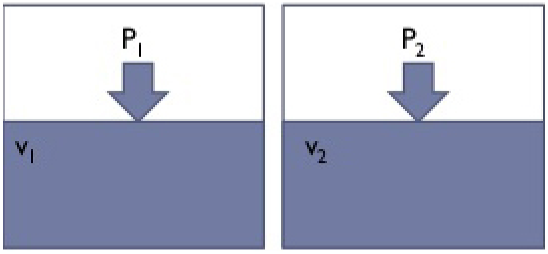

Figure 2.3: Incompressible fluid

(https://m.facebook.com/prepareyourself.gk/photos/what-is-incompressible-fluida-fluid-in-which-the-density-remains-constant-for-is/106071131116820/)

2.3.7 Newtonian fluid

Newtonian fluids are actual fluids that adhere to Newton's viscosity law. for instance, hydrogen and water.

2.4 Boundary layer

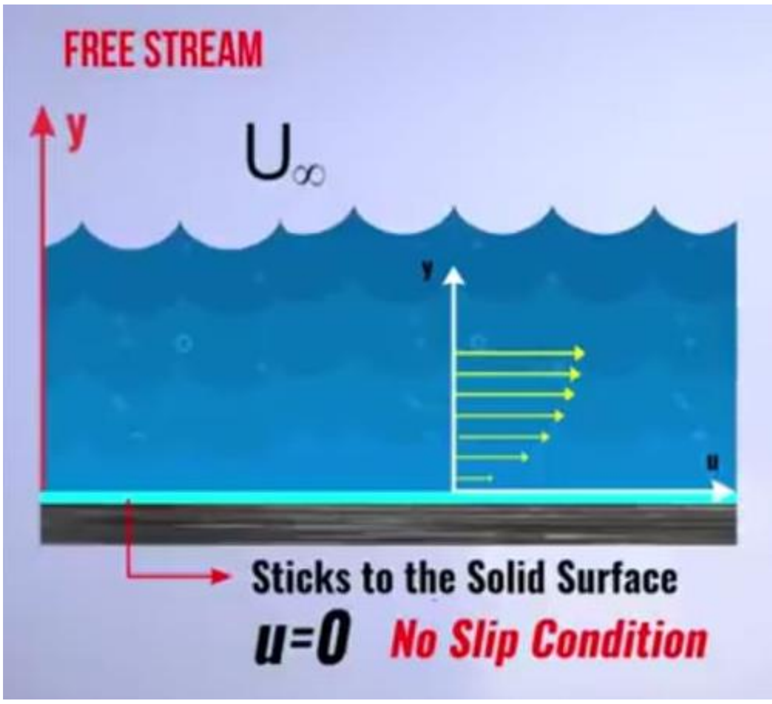

Figure 2.4: Boundary layer

(https://youtu.be/9njAGk_DcFg)

Ludwig Prandtl initially suggested the concept of boundary layers in 1904.According to Prandtl, a body's flow may be separated into two areas:

- The first zone is a thin layer (boundary layer) near to the body where viscosity predominates.
- The second area is outside of this layer, and it is the location where a fluid particle’s velocity reaches 99% of free stream velocity. With respect to distance from the surface velocity will fluctuate for a thin layer.

A thin layer above the surface is known as the boundary layer and has a velocity gradient.

2.5 Heat and mass transfer

A kinetic process, heat transfer involves the movement of particles to transmit energy from one particle to another. On the other hand, mass transfer is the movement of mass from one to another place, as in absorption, evaporation and other processes**.**

2.6 Convection

Heat is transferred through a fluid when it moves across a space, such as heated air or water. Convection develops as a result of most fluids' tendency to expand when they warm up.

2.7 Two-Dimensional flow

Dimensions are essentially space coordinates, and although fluid motions are typically thought of as being three-dimensional, they are often calculated as being two-dimensional in order to make calculations easier. To flow in a plane coordinate is to flow in two dimensions.

2.8 Nanofluids

A fluid in which nanoparticles with a length of 1-100 nm are suspended is referred to as nanofluids. Nanoparticles have a larger potential to improve heat transmission than nanofluids, which can elevate the heat conduction of the liquid.

2.9 Steady flow

At any point in time, the fluid's particle velocity is constant.

Stable flow refers to water moving smoothly through a pipeline with a continuous discharge $\frac{\partial v}{\partial t} = 0.$

2.10 unsteady flow

At any point in time, the fluid's particle velocity is not constant. $\frac{\partial v}{\partial t} \neq 0.$

2.11 Thermal radiation

The process by which energy diffuses across a heat surface in all directions as electromagnetic waves. Energy is transported across material by the process of radiation, either as waves or particles.

2.12 Properties of fluid

2.12.1 Density

The term "density" refers to the number of "things" in a certain amount of space. It is a ratio between mass and volume. It is stated as

$$\rho = \frac{m}{V}.$$

2.12.2 Dynamic viscosity

Dynamic viscosity is defined as the fractional relationship of shear stress to deformation rate,

This is indicated by μ. In mathematics

$$\mu = \frac{shear\ stress}{deformation\ rate}.$$

Dimension = $[L^2T^{-1}]$.

2.12.3 Kinematic viscosity

The fractional relationship of dynamic viscosity to density. It is represented by the letter ν. Kinematic viscosity is expressed mathematically as

$$\nu = \frac{dynamic\ viscosity}{density} = \frac{\mu}{\rho}.$$

2.13 Some useful non-dimensional numbers

2.13.1 Prandtl number

It is a non-dimensional quantity that represents a change in kinematic viscosity v in relation to thermal diffusivity λ. Mathematically

$$Pr = \frac{\nu}{\lambda}.$$

2.13.2 Reynolds number

The ratio of inertial force to the viscous force is called Reynold number. Mathematically it is represented as

$$Re = \frac{\rho v L}{\mu}.$$

2.13.3 Eckert number

Advective mass transfer to heat dissipation potential ratio is known as the Eckert number. It can be stated as follows

$$Ec = \frac{advective\ mass\ transfer}{viscous\ dissipation}.$$

2.13.4 Skin friction

The wall shear stress is used to define the dimensionless skin-friction coefficient:

Mathematically it is indicated by

$$C_f = \frac{\bar{T}_w}{q}.$$

$\bar{T}_w =$ wall shear stress in the area.

q = dynamic free-stream pressure.

2.13.5 Nusselt number

Local Nusselt number is a dimensionless quantity that measures the proportion of conductive to convective heat transfer at the border.

In math, it is represented as

$$Nu_x = \frac{x h_x}{k}.$$

CHAPTER 3

HEAT TRANSMISSION IN A FLUID WITH THERMAL CONDUCTIVITY OVER A STRETCHING SHEET WHICH IS NONLINEAR AND LINEAR.

The given chapter will discuss the transfer of heat in a 2-dimensional Newtonian fluid along the linear and a nonlinear stretching sheet. In order to get to the nonlinear ordinary differential equations (ODEs), stream operation was explained in a versatile way in this case (comparatively to the linear stretching case). These differential equations are numerically solved, given the boundary conditions. The governing partial differential equations (PDEs) will be transformed into dimensionless ordinary differential equations (ODEs). To solve the ordinary differential equation, numerical method is used. This chapter is the review of an article of Vajravelu, K. [**39**] for the boundary layer flow along a nonlinear stretching sheet and an article of Chaim, T. [**40**] for boundary layer flow along linearly stretching sheet.

3. 1 Mathematical Formulation:

Consider a 2-dimensional flow, incompressible steady flow of a viscous fluid along nonlinear stretching sheet having velocity $u = cx^n$. Here linear $n = 1$ and non-linear $n > 1$.The equations of continuity, velocity and temperature are given as below.

$$\nabla . V = 0, \tag{3.1}$$

$$\rho \, (V.\nabla)V = div\tau, \tag{3.2}$$

$$\left(\rho C_p\right)(V.\nabla)T = \tau.(\nabla.V) - divq, \tag{3.3}$$

Where τ can be expressed as

$$\tau = -pI + \mu L_1, \tag{3.4}$$

Here L_1 is Rivlin-Ericksen tensor, which can be written as

$$L_1 = (\nabla V) + (\nabla V)^T, \tag{3.5}$$

The respective Velocity and temperature field will be

$$V = [u(x,y), v(x,y), 0], \qquad T = T(x,y), \tag{3.6}$$

Using Equation (3.6)in Equation (3.5) and we get

$$\nabla V = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, (\nabla V)^T = \begin{bmatrix} u_x & v_x & 0 \\ u_y & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3.7}$$

Now by utilizing Equation (3.7) in (3.5), we obtain

$$L_1 = \begin{bmatrix} 2u_x & u_y + v_x & 0 \\ v_x + u_y & 2v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3.8}$$

Putting the value of Equation (3.8) in Equation (3.4)

$$\tau = \begin{bmatrix} -p + 2\mu u_x & \mu(u_y + v_x) & 0 \\ \mu(v_x + u_y) & -p + 2\mu v_y & 0 \\ 0 & 0 & -p \end{bmatrix}, \tag{3.9}$$

To express equation (3.9) in component form that is

$$\tau_{xx} = -p + 2\mu u_x,\ \tau_{xy} = \tau_{yx} = \mu\left(v_x + u_y\right), \tag{3.10}$$

$$\tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0, \tag{3.11}$$

$$\tau_{yy} = -p + 2\mu v_y, \tau_{zz} = -p, \tag{3.12}$$

Putting equations (3.10) , (3.11) and (3.12)in equation (3.2)

$$\rho\left(uu_x + vu_y\right) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u, \tag{3.13}$$

$$\rho\left(uv_x + vv_y\right) = -\frac{\partial p}{\partial y} + \mu\nabla^2 v, \tag{3.14}$$

Now, using equation

$$q = -k\left[T_x, T_{y,} 0\right], \tag{3.15}$$

$$\rho C_p\left(uT_x + T_y\right) = k\nabla^2 T, \tag{3.16}$$

$$\rho C_p\left[u\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y}\right] = k\left[\frac{\partial^2 T}{\partial x^2}\right]. \tag{3.17}$$

The boundary condition for equations are

$$u = u_w(x) = cx^n \, , v = 0, \; T = T_w \qquad \text{at} \quad y = 0, \tag{3.18}$$

$$u \to 0 \, , \qquad T \to T_\infty \qquad \text{as} \quad y \to \infty. \tag{3.19}$$

in which T_w and u_w respectively represent the temperature and velocity of stretching boundary. Here we studied two cases for the stretching velocity i.e., Nonlinear stretching and linearly stretching.

Case:1 $n > 1$

For nonlinear velocity

Case:2 $n = 1$

For linear velocity

To convert above non-linear partial differential equations (PDEs) to linear ordinary differential equations (ODEs) will consider the following transformation(similarity).

$$\eta = y\sqrt{c(n+1)/2\nu}x^{(n-1)/2}, \tag{3.20}$$

$$\psi = \sqrt{c\nu}\, x^n \, f(\eta) \; , \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{3.21}$$

Converting ψ into u and v

$$u = \frac{\partial \psi}{\partial y} = cx^n f'(\eta), \tag{3.22}$$

$$v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{C\nu(n+1)}{2}} x^{\frac{n-1}{2}} \left[f + \left(\frac{n-1}{n+1}\right)\eta f' \right], \tag{3.23}$$

Now using above similarity transformations in Equations (3.14) and (3.15) respectively,

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)(f'^2) = 0, \tag{3.24}$$

$$\theta'' + Prf\theta' = 0. \tag{3.25}$$

Where $Pr = \frac{\mu c_p}{k}$.

Depressed boundary conditions are

$$f(0) = 0, \; f'(0) = 1, \; \theta(0) = 1, \qquad at \;\; \eta = 0, \tag{3.26}$$

$$f' \to 0, \qquad \theta \to 0 \qquad\qquad as\, \eta \to \infty. \tag{3.27}$$

Skin friction and Nusselt number are denoted by the following expressions.

$$C_f = \frac{2\bar{T}_w}{\rho u^2{}_w}, \tag{3.28}$$

Where,

$$\bar{T}_w = \mu \frac{\partial u}{\partial y} \quad at\; y = 0. \tag{3.29}$$

The Nusselt number is

$$Nu_x = \frac{x q_w}{K(T_w - T_\infty)}, \tag{3.30}$$

Heat flux is represented by q_w

$$q_w = -K\left(\frac{\partial T}{\partial y}\right) \qquad at\; y = 0. \tag{3.31}$$

Now utilizing comparable (similarity) variables Equations (3.20), (3.21) and (3.22) in Equations (3.28) and (3.30) then we get dimensionless skin fiction and Nusselt number respectively.

Case; I

For nonlinear stretching sheet by substituting $n > 1$, then we get

$$f''(0) = C_f \sqrt{\frac{Re_x}{2(n+1)}}, \tag{3.32}$$

$$-\theta'(0) = Nu_x \sqrt{\frac{2}{Re_x(n+1)}}. \tag{3.33}$$

Case: II

For linearly stretching sheet by substituting $n = 1$ in Equations (3.32) and (3.33) then we get

$$f''(0) = \frac{Cf}{2}\sqrt{Re_x}, \tag{3.34}$$

$$-\theta'(0) = \frac{Nu\_x}{\sqrt{Re_x}}. \tag{3.35}$$

Here $Re_x = \frac{U_w x}{\upsilon}$ is Reynold number.

f' and θ' represent the differentiate with respect to (w.r.t) η.

3.2 Methodology:

The ordinary differential equations (ODEs) Equations (3.24) and (3.25) with the accompanying boundary conditions (3.26) and (3.27) can be solved using the shooting procedure and Runge-Kutta method. In order to solve the system (3.24), (3.25) it is required to be translated into a first order initial value problem (IVP). So that we take

$$f = b(1), \qquad \theta = b(4), \tag{3.2.1}$$

$$f' = b(2), \qquad \theta' = b(5), \tag{3.2.2}$$

$$f'' = b(3), \qquad \theta'' = bb2, \tag{3.2.3}$$

$$f''' = bb1, \tag{3.2.4}$$

Equation (3.24) $\Rightarrow bb1 = -b(1)b(3) + \frac{2n}{n+1}b(2)^2;$ (3.2.5)

Equation(3.25)$\Rightarrow bb2 = Pr(-b(1)b(5);$ (3.2.6)

$$\Rightarrow \begin{bmatrix} f' \\ f'' \\ f''' \\ \theta' \\ \theta'' \end{bmatrix} = \begin{bmatrix} b(2) \\ b(3) \\ bb1 \\ b(5) \\ bb2 \end{bmatrix} = \begin{bmatrix} b(2) \\ b(3) \\ -b(1)b(3) + \frac{2n}{n+1}b(2)^2 \\ b(5) \\ Pr(-b(1)b(5) \end{bmatrix}. \tag{3.2.7}$$

Here are the initial conditions:

$$b(1)(0) = 0\,,\ b(2)(0) = 1\,,\ b(4)(0) = 1, \tag{3.2.8}$$

$$b(2)(\infty) = 0,\ b(4)(\infty) = 0. \tag{3.2.9}$$

3.3 Result and discussion:

The nonlinear ODEs are solved in MATLAB using shooting method. The transformed system of the said ODEs (3.24) and (3.25) with specific boundary conditions (3.26) and (3.27) are solved numerically. Moreover, these numerical solutions are concluded over some values of governing parameters. The given figures **3.1**, **3.2** and **3.3** illustrates the significant traits of the flow and head of transfer characteristics. These graphs display the temperature profiles and velocity profiles. We examine the dominant parameters including temperature, velocity and the Prandtl number Pr.

Figures **3.1** and **3.2** illustrate the velocity profile $f\ '(\eta)$ and temperature profile $\theta(\eta)$ for three nonlinear stretching parameters n (1, 5 and 10) when $Pr = 0.71$. Figure **3.1** demonstrated that the velocity $f'(\eta)$ reduces as n increases. Additionally, this drop in $f'(\eta)$ is insignificant for large value of n. This result from the fact that when n approaches to infinity, the coefficient $\frac{2n}{n+1}$ in the differential equation (3.24) approaches 2. The value of n has a significant impact on the velocity component u an v.

The non-dimensional temperature depicted in Figure **3.2** for some value of n for Prandtl number $Pr = 0.71$. Figure **3.2** shows clearly that the temperature $\theta(\eta)$ rises as the nonlinear stretching parameter n is increased.

Our publications have time and again confirmed the precision and validity of the equations with boundary conditions for linearly stretching sheet.

Figure **3.3** makes it evident that the Prandtl number is in inverse relationship with the thickness of thermal boundary layer. Resultantly, curve of the graph becomes steep.

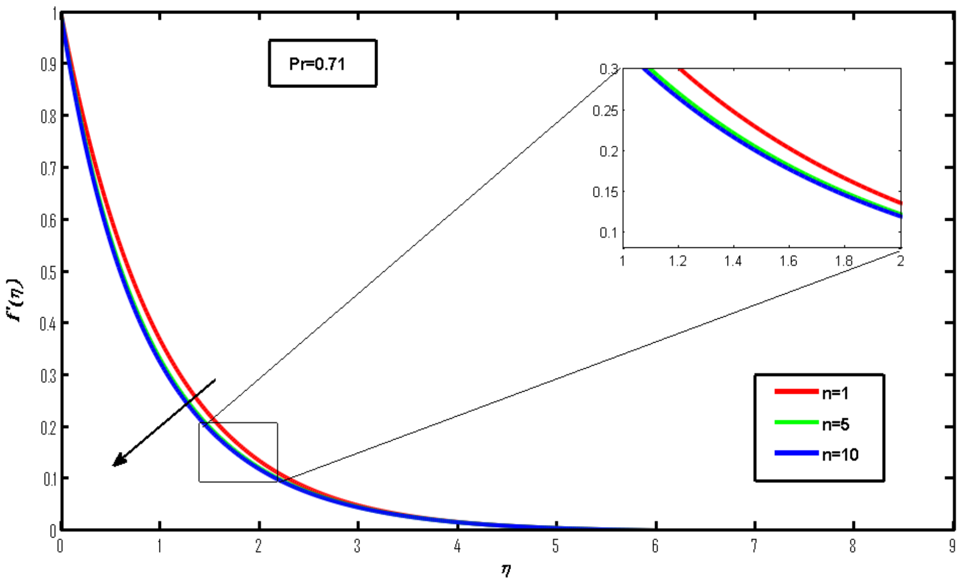

Figure 3.1: Fluctuation of velocity f' in relation to n

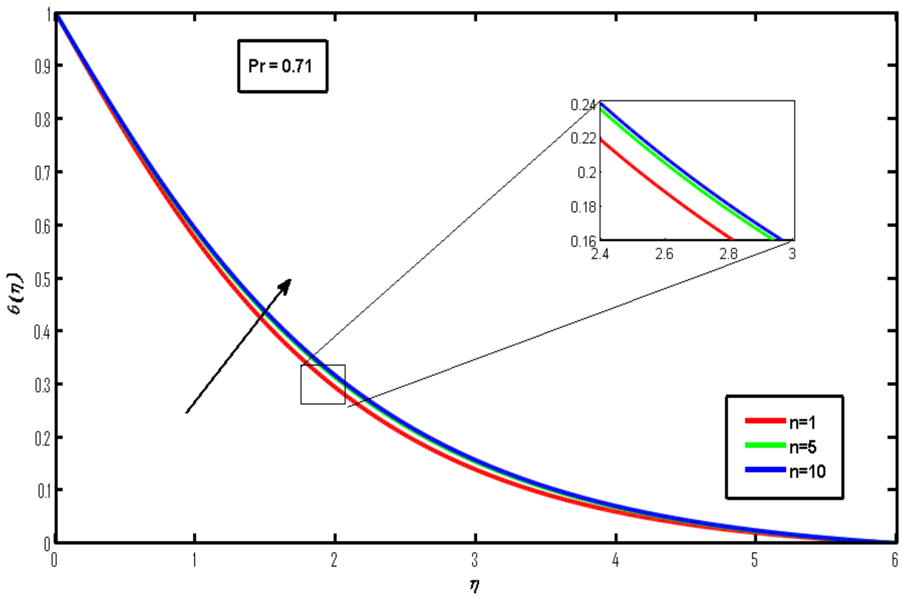

Figure 3.2: Fluctuation of the temperature $\theta(\eta)$ in relation to n

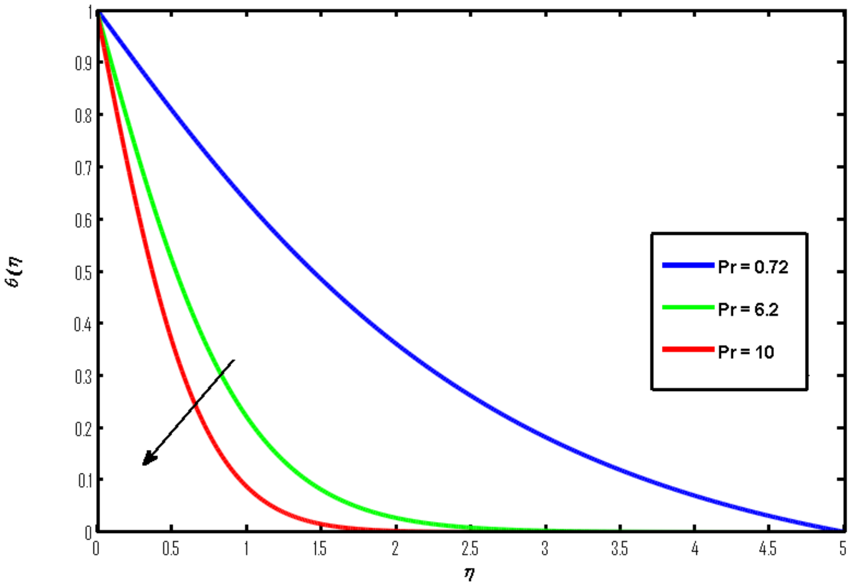

Figure 3.3: Fluctuation of the temperature $\theta(\eta)$ when $n = 1$

CHAPTER 4

FLOW OF AN EXPONENTIAL STRETCHING SHEET WITH THERMAL RADIATION USING NUMERICAL SOLUTION.

This chapter cover heat transmission in a two-dimensional, incompressible, viscous liquid when thermal radiation is present along a sheet that is stretching exponentially. The governing partial differential equations (PDEs) will be translated into dimensionless ordinary differential equations (ODEs). Numerical method is utilized to resolve the underlying issues of ordinary differential equation (ODE). The given chapter reviews [**41**].

4.1 Mathematical formulation:

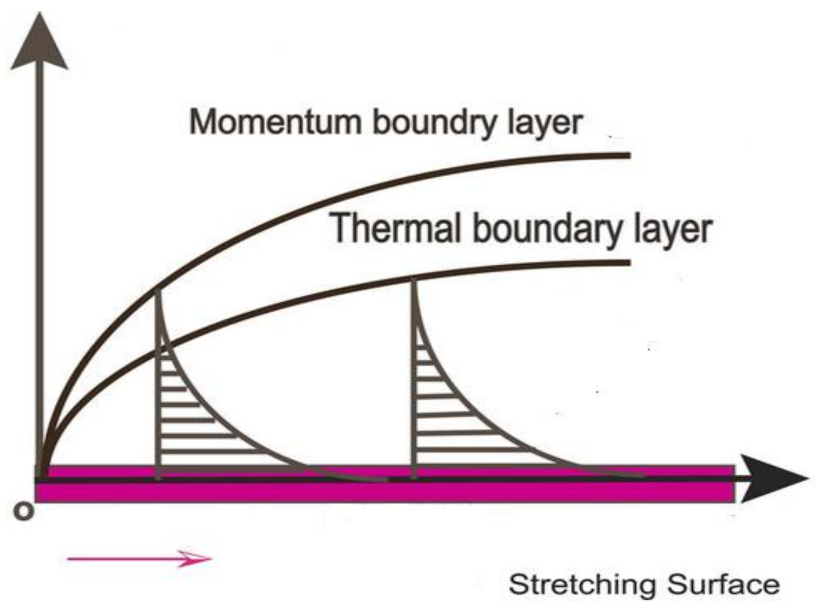

Figure 4.1: Boundary layer flow

Consider a 2-dimensional flow, incompressible steady viscous fluid flow along a nonlinearly stretching sheet having velocity $u = U_0 e^{\frac{x}{L}}$.The governing equations of continuity, velocity and temperature are Equations (4.1 – 4.3).

$$\nabla . V = 0, \tag{4.1}$$

$$\rho\ (V.\nabla)V = div\tau, \tag{4.2}$$

$$\left(\rho C_p\right)(V.\nabla)T = \ \tau.(\nabla.V) - divq, \tag{4.3}$$

Here τ is represented by

$$\tau = -pI + \mu L_1, \tag{4.4}$$

Here L_1 is Rivlin-Ericksen tensor is expressed as:

$$L_1 = (\ \nabla\mathrm{V}) + (\nabla V)^T, \tag{4.5}$$

The respective Velocity and temperature field will be:

$$V = [u(x,y), v(x,y), 0], \qquad T = T(x,y), \tag{4.6}$$

Using Equation (4.6) in Equation (4.5) and we get

$$\nabla V = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, (\nabla V)^T = \begin{bmatrix} u_x & v_x & 0 \\ u_y & v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4.7}$$

Now by utilizing Equation (4.7) in (4.5), we obtain

$$L_1 = \begin{bmatrix} 2u_x & u_y + v_x & 0 \\ v_x + u_y & 2v_y & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4.8}$$

Substituting Equation (4.8) in Equation (4.4), it results

$$\tau = \begin{bmatrix} -p + 2\mu u_x & \mu(u_y + v_x) & 0 \\ \mu(v_x + u_y) & -p + 2\mu v_y & 0 \\ 0 & 0 & -p \end{bmatrix}, \tag{4.9}$$

To express Equation (4.9) in component form that is

$$\tau_{xx} = -p + 2\mu u_x,\ \tau_{xy}\ =\ \tau_{yx} = \mu\left(v_x + u_y\right), \tag{4.10}$$

$$\tau_{xz}\ =\ \tau_{zx} = \tau_{yz}\ =\ \tau_{zy} = 0, \tag{4.11}$$

$$\tau_{yy} = -p + 2\mu v_y, \tau_{zz} = -p, \tag{4.12}$$

Putting Equations (4.10, 4.11) and (4.12) in Equation (4.2)

$$\rho\left(uu_x + vu_y\right) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u, \tag{4.13}$$

$$\rho\left(uv_x + vv_y\right) = -\frac{\partial p}{\partial y} + \mu\nabla^2 v, \tag{4.14}$$

Now, using Equation

$$q = -k\left[T_x, T_{y,} 0\right], \tag{4.15}$$

$$\rho C_p\left(uT_x + T_y\right) = k\nabla^2 T + \mu(\nabla u)^2 - \nabla q_r, \tag{4.16}$$

$$\rho C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left[\frac{\partial^2 T}{\partial y^2}\right] + \mu\left[\frac{\partial u}{\partial y}\right]^2 - \left[\frac{\partial q_r}{\partial y}\right]. \tag{4.17}$$

The boundary conditions for equations are:

$$u = u_w = U_0 e^{\frac{x}{L}}, v = 0,\ T = T_w = T_\infty + T_0 e^{\frac{x}{2L}} \quad \text{at} \quad y = 0, \tag{4.18}$$

$$u \to 0, \qquad T \to T_{\infty,} \qquad \text{as} \quad y \to \infty. \tag{4.19}$$

in which T_w and u_w respectively represent the temperature and velocity of stretching boundary.U_O represent source velocity and L is constant.

Here,

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{4.20}$$

In which $k^*$and σ^* refer to mean consumption coefficient and the constant of Stefan-Boltzmann respectively. T^4 is a linear function.

Here,

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \tag{4.21}$$

To convert above non-linear partial differential equations (PDEs) to linear ordinary differential equations (ODEs) will consider following transformation(similarity):

$$\eta = \sqrt{\frac{U_0}{2\upsilon L}}\, e^{\frac{x}{2L}} y, \quad T = T_0 e^{\frac{x}{2L}}\theta(\eta), \tag{4.22}$$

Converting ψ into u and v

$$u = U_0 e^{\frac{x}{L}} f'(\eta), \tag{4.23}$$

$$v = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \{f(\eta) + \eta f'(\eta)\}, \tag{4.24}$$

Now using above similarity transformations, the respective equations become:

$$f''' - 2f'^2 + ff'' = 0, \tag{4.25}$$

$$\left(1 + \frac{4}{3}K\right)\theta'' + Pr[f\theta' - f'\theta + Ef''^2] = 0. \tag{4.26}$$

With respective boundary conditions:

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1, \quad at\ \eta = 0, \tag{4.27}$$

$$f' \to 0,\ \theta \to 0 \qquad\qquad as\ \eta \to \infty. \tag{4.28}$$

Where Prandtl number Pr, Eckert number E and Radiation number K expressed as,

$$Pr = \frac{\mu c_p}{k},\ E = \frac{U_O^2}{T_0 c_p}\ ,\ K = \frac{4\sigma^* T_\infty^3}{k^* k}. \tag{4.29}$$

Skin friction C_f can be written as:

$$C_f = \frac{2\bar{T}_w}{\rho u^2{}_w}, \tag{4.30}$$

Where,

$$\bar{T}_w = \mu \frac{\partial u}{\partial y} \quad at\ y = 0. \tag{4.31}$$

The Nusselt number is

$$Nu_x = \frac{x q_w}{K(T_w - T_\infty)}, \tag{4.32}$$

Heat flux is represented by q_w

$$q_w = -K\left(\frac{\partial T}{\partial y}\right) \qquad at\ y = 0. \tag{4.33}$$

Now utilizing comparable (similarity) variables Equations (4.22, 4.23) and (4.24) in Equations (4.30) and (4.32) then we get dimensionless skin fiction and Nusselt number respectively.

$$f''(0) = \frac{1}{\sqrt{2}}\sqrt{Re_x}C_f, \tag{4.34}$$

$$-\theta'(0) = \frac{Nu_x}{\sqrt{Re_x}}. \tag{4.35}$$

Here $Re_x = \frac{U_w x}{\upsilon}$ is Reynold number.

4.2 Methodology:

The ordinary differential equations (ODEs) (4.25) and (4.26) with the accompanying boundary conditions (4.27) and (4.28) can be solved using the shooting technique and Runge-Kutta method. In order to addressed the problem of the given system (4.25) and (4.26) it is required to be transformed into a first order initial value problem (IVP). So that we take

$f = s(1), \qquad \theta = s(4),$ (4.2.1)

$f' = s(2), \qquad \theta' = s(5),$ (4.2.2)

$f'' = s(3), \qquad \theta'' = ss2,$ (4.2.3)

$f''' = ss1,$ (4.2.4)

Equation (4.25) ⇒ $ss1 = -s(1)s(3) + 2s(2)^2;$ (4.2.5)

Equation (4.26) ⇒ $ss2 = -\frac{1}{1+\frac{4k}{3}} Pr(s(1)s(5) - s(2)s(4) + Es(3)^2);$ (4.2.6)

$$\Rightarrow \begin{bmatrix} f' \\ f'' \\ f''' \\ \theta' \\ \theta'' \end{bmatrix} = \begin{bmatrix} s(2) \\ s(3) \\ ss1 \\ s(5) \\ ss2 \end{bmatrix} = \begin{bmatrix} s(2) \\ s(3) \\ -s(1)s(3) + 2s(2)^2 \\ s(5) \\ -\frac{1}{1+\frac{4k}{3}} Pr(s(1)s(5) - s(2)s(4) + Es(3)^2) \end{bmatrix}. \quad (4.2.7)$$

Here are the initial conditions:

$s(1)(0) = 0\,,\ s(2)(0) = 1\,,\ s(4)(0) = 1,$ (4.2.8)

$s(2)(\infty) = 0,\ s(4)(\infty) = 0.$ (4.2.9)

4.3 Result and discussion:

Analytical solutions are obtained over the behavior of highly nonlinear boundary layer through exponentially stretching sheet. The main factor under the observation is the viscoelastic fluid flow over the said (exponentially) stretching sheet. During solution derivation, partial differential equations (PDEs) were reshaped into ordinary differential equations (ODEs). The procedure required suitable similarity transformation We analyze the prevailing factors Prandtl number Pr, Eckert number E, Reynold number Re, thermal radiation K, temperature profile $\theta(\eta)$ and velocity profile $f'(\eta)$.

Figure **4.2** show how the Eckert number $E = 0.2$, radiation parameter $K = 1.0$ and Prandtl number $Pr = 1.0$ effects the temperature profile $\theta(\eta)$ and the profile of velocity $f'(\eta)$. Due to the decoupled Equations (4.25) and (4.26) the velocity profile is distinct for all values of E, K and Pr. It is demonstrated that the relationship between the velocity $f'(\eta)$. and temperature profile $\theta(\eta)$ is linear. Figures **4.3**, **4.4** and **4.5** shows how the temperature profile $\theta(\eta)$ is affected by the Pr, E, K are Prandtl number, Eckert number and radiation parameter respectively.

Figure **4.3** illustrated how a rise in the Prandtl number Pr results in a fall in the temperature profile $\theta(\eta)$ and thickness of thermal boundary layer with radiation parameter $K = 1.0$ and Eckert number $E = 0.2$. Physically as Prandtl number Pr rises, the thermal diffusivity falls and these events result in a decline in energy capacity, which resultantly lowers the thermal boundary layer. On the other hand, Figure **4.4** expresses the behavior of temperature profile $\theta(\eta)$ and the thickness of thermal boundary layer both slightly rise when Eckert number E increases and Prandtl number $Pr = 1.0$ and radiation parameter $K = 1.0$ are stable. Figure **4.5** illustrate the behavior of temperature over variation of K with. Prandtl number $Pr = 1.0$ and Eckert number $E = 0.2$. It is evident that as K increases temperature profile $\theta(\eta)$ also rises.

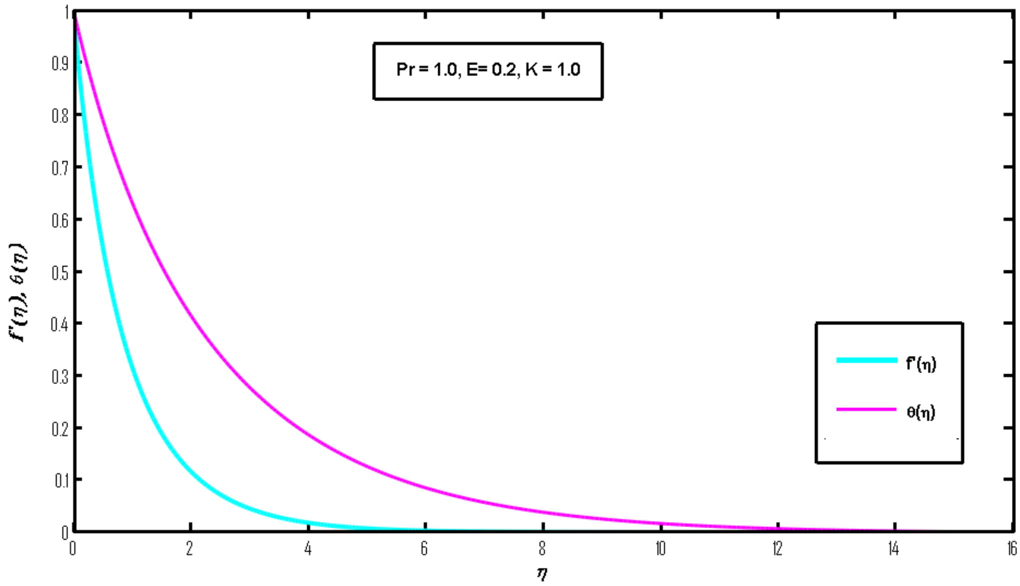

Figure 4.2: Velocity profile$f'(\eta)$ and temperature $\theta(\eta)$

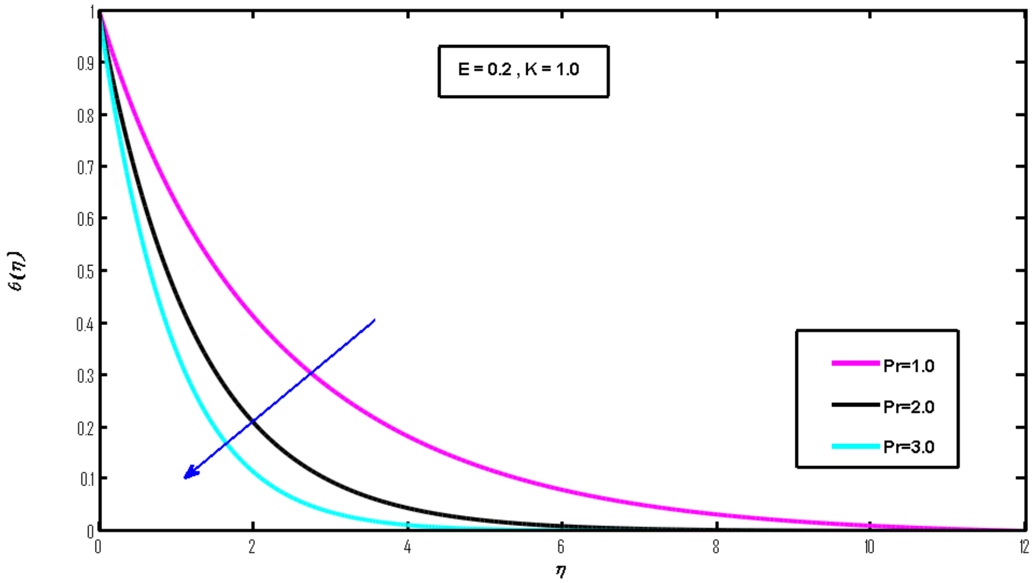

Figure 4.3: Impact of Prandtl number Pr on the temperature profile $\theta(\eta)$

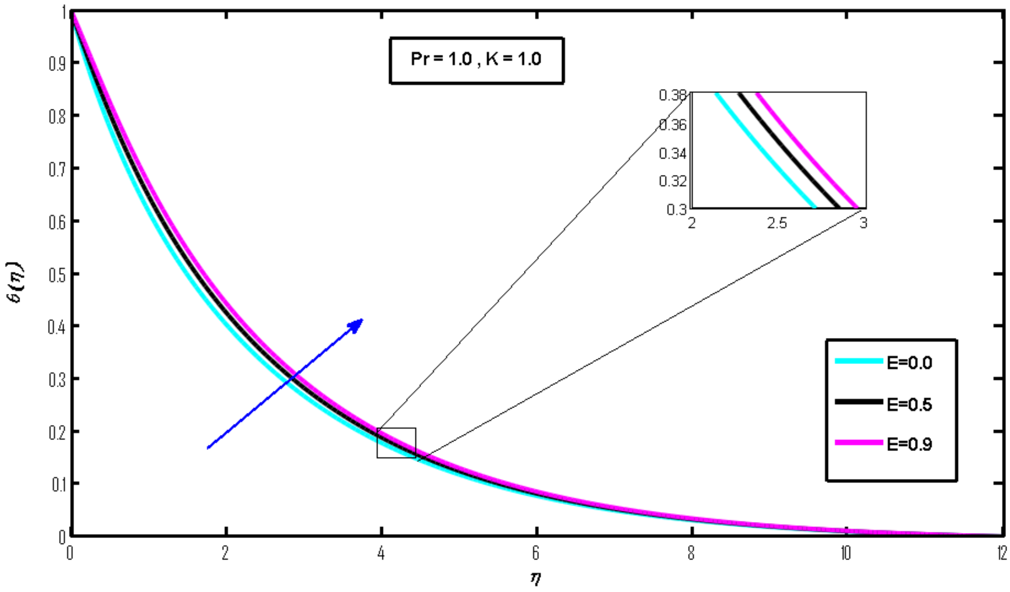

Figure 4.4: Impact of Eckert number E on the temperature profile $\theta(\eta)$

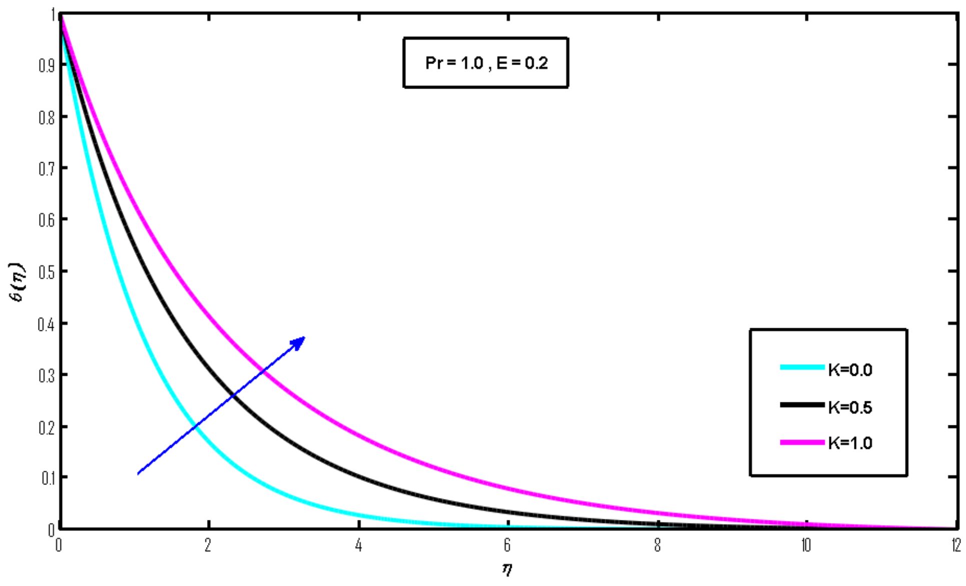

Figure 4.5: Radiation number's impact on the temperature profile $\theta(\eta)$

CHAPTER 5

RELATION AMONG FLUID FLOW OVER A LINEAR, NONLINEAR AND EXPONENTIALLY STRETCHING/SHRINKING SHEETS.

This chapter an enhanced version of my work a graphical comparison of linear, nonlinear and exponential stretching/shrinking sheet is made and combined as one to show the velocity and temperature fluid flow. The results are obtained by numerical solution. Subsequently, mathematical formulation was modeled using boundary conditions. All partial differential equations (PDEs) of energy and momentum are converted into nonlinear ordinary differential equations (ODEs) for a formal formulation. This is done with the help of similarity variables. At the end, nonlinear ordinary differential equations (ODEs) are solved by using MATLAB software.

5.1 Mathematical Formulation:

Consider 2-D steady fluid flow bounded by a stretched sheet. Where $u = cx^n$ (c is positive constant) is the velocity of stretching/shrinking sheet. For $n = 1$ it is linear and for $n > 1$ it is nonlinear. Where the exponential stretching sheet with $u = ce^{\frac{x}{L}}$. The basic boundary layer governing equations as follows:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \tag{5.1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial y}\right) = \mu\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right), \tag{5.2}$$

$$(\rho C_p)\ \left(\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right) = \kappa\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right). \tag{5.3}$$

The boundary conditions for stretching/shrinking sheet:

For nonlinearly stretching /shrinking sheet

$$u = u_w(x) = ax^n,\ n > 1\ , v = 0,\ T = T_w \qquad \text{at} \quad y = 0\,, \tag{5.4}$$

$$u \to 0,\ \ T \to T_\infty, \qquad \text{as} \quad y \to \infty. \tag{5.5}$$

For linearly stretching /shrinking sheet

$$u = u_w(x) = ax,\ \ , v = 0,\ T = T_w \qquad \text{at} \quad y = 0\ , \tag{5.6}$$

$$u \to 0,\ \ T \to T_\infty, \qquad \text{as} \quad y \to \infty. \tag{5.7}$$

For exponentially stretching /shrinking sheet

$$u = u_w = ae^{\frac{x}{L}}, u = v = 0,\ T = T_w = T_\infty + T_0 e^{\frac{x}{2L}} \ \text{at} \quad y = 0, \tag{5.8}$$

$$u \to 0\ \ ,\ \ T \to T_\infty, \qquad \text{as} \quad y \to \infty\ . \tag{5.9}$$

Determined by Equations (5.4 - 5.9) along with the relevant boundary condition

For nonlinearly/linearly stretching /shrinking sheet

$$f(0) = 0,\ f'(0) = \lambda,\ \theta(0) = 1, \qquad at \ \ \eta = 0, \tag{5.10}$$

$$f' \to 0, \qquad \theta \to 0\ , \qquad as\ \eta \to \infty\ . \tag{5.11}$$

For exponentially stretching /shrinking sheet

$$f(0) = 0,\ f'(0) = \lambda,\ \theta(0) = 1, \quad at\ \eta = 0, \tag{5.12}$$

$$f' \to 0,\ \theta \to 0\ \ , \qquad as\ \eta \to \infty\ . \tag{5.13}$$

In this case, primes signify differentiation in relation to η.

5.2 Result and discussion:

The main objective of this research is making comparison of mass and heat transfer characteristics of viscous flow due to sheet stretching/shrinking at linear, nonlinear and exponential rates. The process involves the conversion of partial differential equations (PDEs) into ordinary differential equations (ODEs) to give graphical solution. This chapter introduces new parameters such as skin friction C_f , Reynold number Re_x, Nusselt number Nu_x, stretching/shrinking parameter λ , thermal radiation K, Prandtl number Pr, temperature profile $\theta(\eta)$, velocity profile $f'(\eta)$ and Eckert number E_c. It represents the numerical solutions and graphical comparisons of all three dimensions.

Figure **5.1** is plotted to predict the behavior of velocity $f'(\eta)$ for linear, nonlinear and exponentially stretching sheet. Our aim is to compare the velocity profile generated due to different stretching velocities. The graph shows that the exponentially stretching sheet produces lower value of flow than that of linear, nonlinear stretching velocities. Also, the linear velocity for $n = 1$ has greater value comparative to nonlinear stretching sheets i.e., for $n = 5$.The most significant feature of this graph is the unchanging behavior of velocity for different values of Prandtl number Pr. It shows that the velocity remains unaffected even for varying and increasing Prandtl number. The figure **5.2** represents a graph that is created to analyze the behavior of temperature $\theta(\eta)$ over linear, nonlinear and exponential stretching sheet. The graph shows a descending order of exponential, nonlinear and linear stretching sheet. Exponential sheet sits on the top with respect to the values of linear for $n = 1$ and nonlinear for $n = 5$ follows it.

Figures **5.3** and **5.4** shows the impact of physical parameter on dimensionless skin friction coefficient C_f and Nusselt number Nu_x. In Figure **5.3** indicates of that for the larger values of Prandtl number Pr skin friction is constant. For particular Prandtl number Pr the exponential stretching sheet shows highest values. The graph shows in ascending order i.e., linear, nonlinear, and exponential stretching sheet with respect to particular value of Prandtl number Pr in skin friction. Consequently, it can be said that the increase in Prandtl number Pr does not cause any variation in skin friction. In the absence of thermal radiation in Figure **5.4** owing to the rise in Prandtl number Pr

Nusselt number Nu_x decreases gradually. For Prandtl number the linearly stretching sheet has higher values than nonlinear and exponential sheet.

Figure **5.5** compares the dual velocity of stretching and shrinking behavior of linear, nonlinear and exponential sheet over of the particular stretching/shrinking parameter λ.The velocity profile for linear, nonlinear and exponential stretching and shrinking sheet converges. With reference to the figure for $\lambda = 7$ create the upper branch and for $\lambda = -7$ creates the lower branch. The Figure **5.6** depicts the plotting of velocity profile $f'(\eta)$ for some value of stretching parameter $\lambda > 0$.It can be seen that the solution of momentum boundary layer is getting thinner for linear, nonlinear and exponential stretching sheet. Figure **5.7** predicts the behavior of velocity profile for different value of shrinking parameter $\lambda < 0$ as in Figure **5.6** but in reverse flow. Initially, the velocity decreases until it become negative. Then for bigger values of η it becomes greater than zero (positive). In Figures **5.6** and **5.7** a reverse flow is witnessed when stretching and shrinking velocities are plotted in opposite direction.

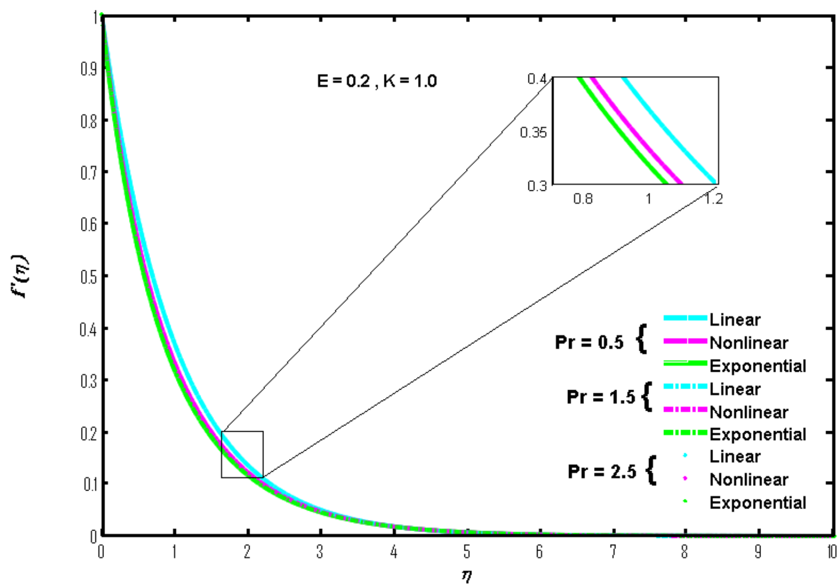

Figure 5.1: Influence of Pr on velocity $f'(\eta)$

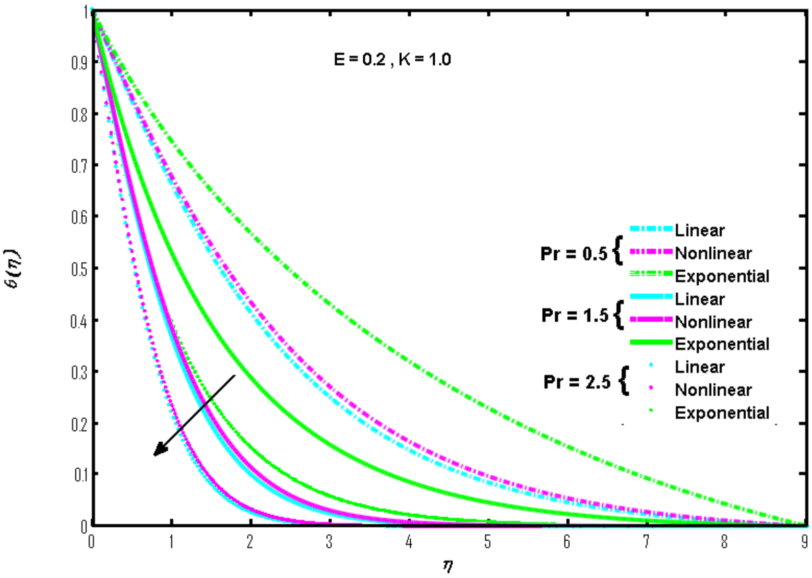

Figure 5.2: Influence of Pr on temperature $\theta(\eta)$

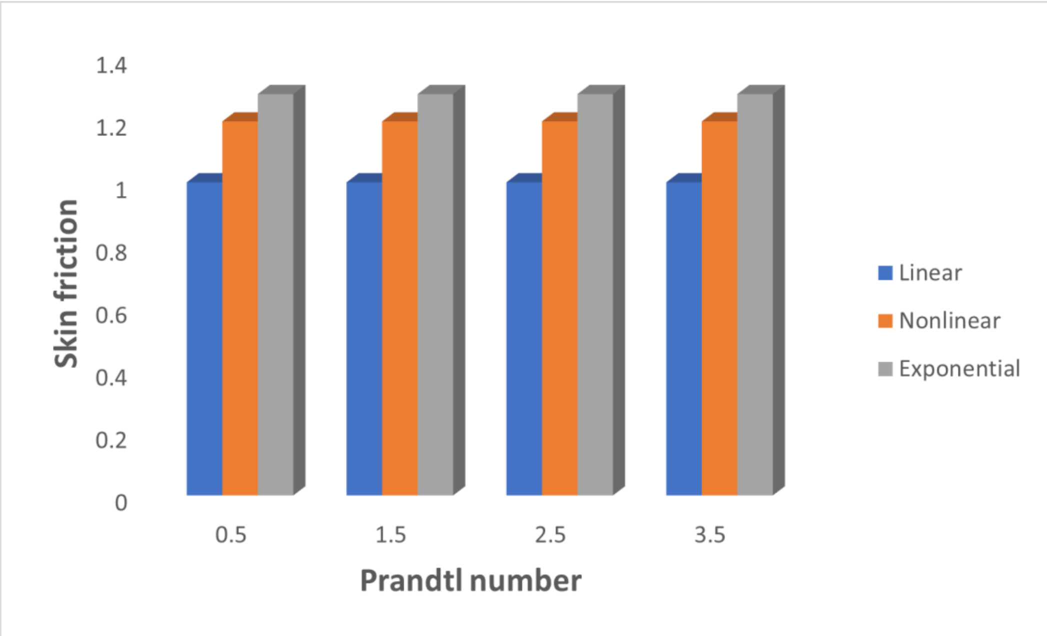

Figure 5.3: Influence of skin friction c_f for Prandtl number Pr

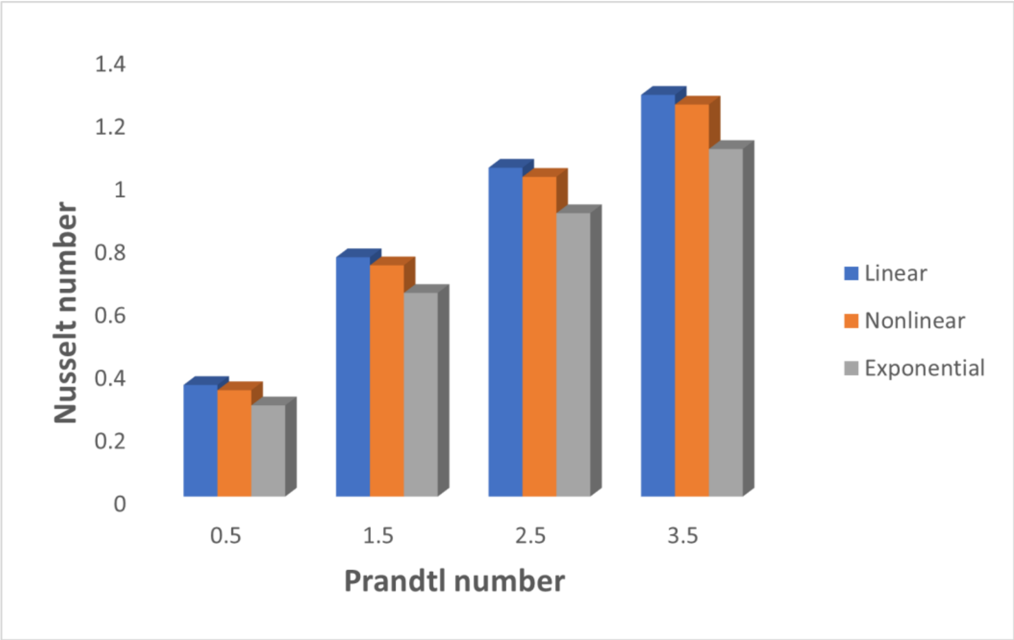

Figure 5.4: Influence of Nusselt number Nu_x for Prandtl number Pr

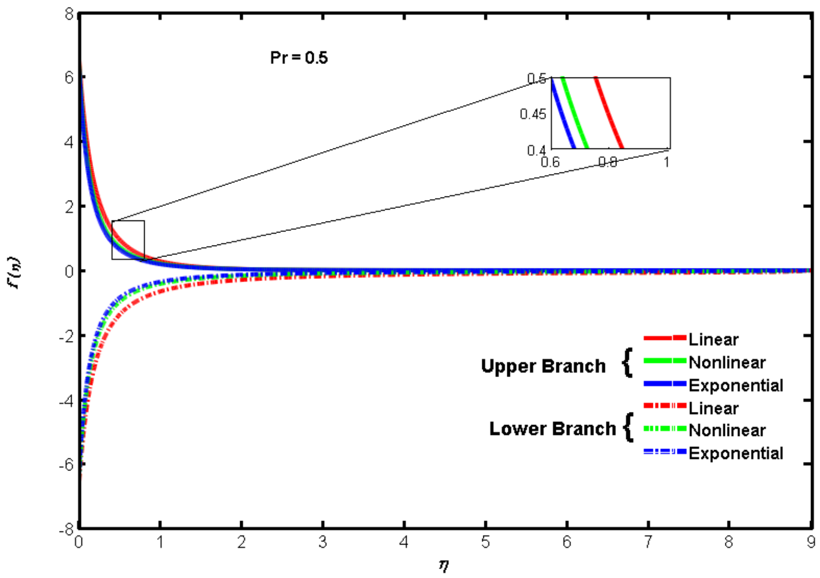

Figure 5.5: velocity profile $f'(\eta)$ for stretching/shrinking parameter λ

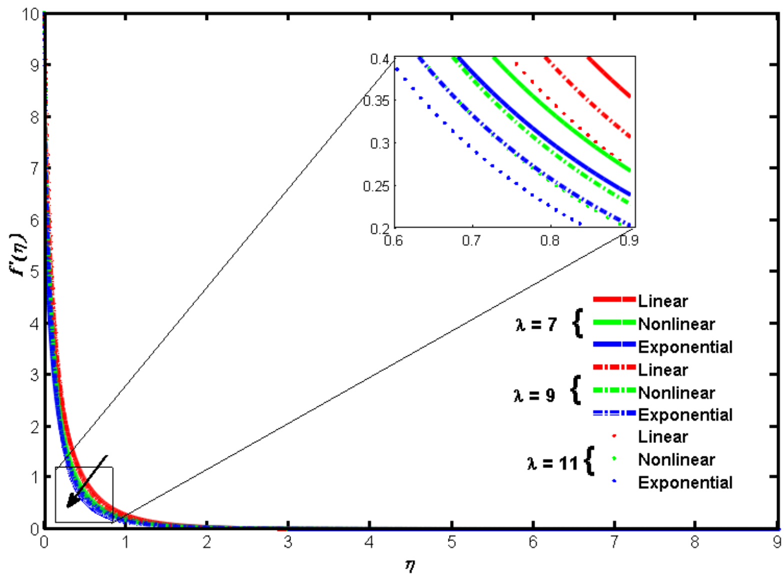

Figure 5.6: Velocity profile changing with stretching parameter λ

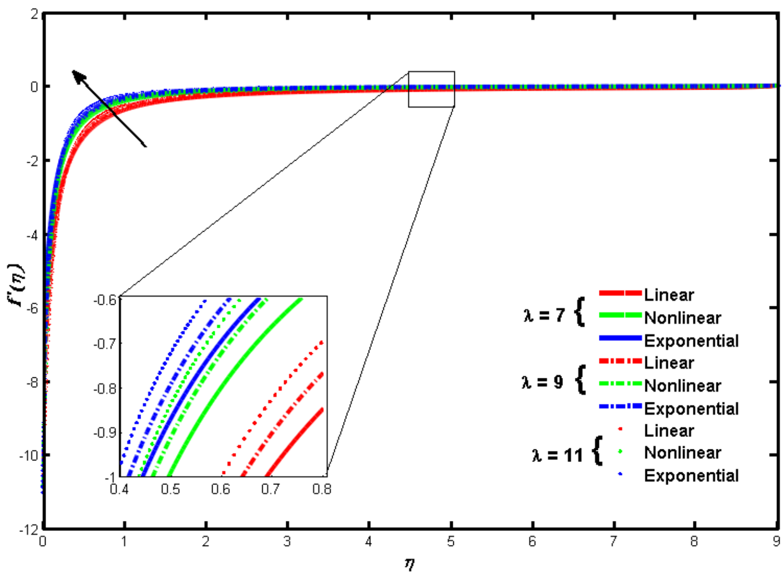

Figure 5.7: Velocity profile changing with shrinking parameter λ

CHAPTER 6

CONCLUSION

This section contains findings about relationship of fluid among linear, nonlinear and exponentially stretching sheets. This chapter sum up all results from preceding three papers.

- The partial differential equations (PDEs) of energy and momentum are converted into nonlinear dimensionless ordinary differential equations (ODEs) with respect to boundary conditions.
- The governing PDEs translated into ODEs using similarity variables for a numerical solution.
- A graphical comparison of linear, nonlinear and exponential stretching sheet is made and combined as one to show the velocity and temperature fluid flow.
- This research work involves non dimensional parameters such as thermal radiation K, skin friction C_f, Nusselt number Nu_x, Eckert number E and Prandtl number $Pr.$
- The main tool utilized to figure out the numerical solution of linear, nonlinear and exponentially stretching /shrinking sheet is MATLAB using bvp4c shooting method.
- The thermal boundary layer thickness is directly proportional (increases) to the Prandtl number of linear, nonlinear stretching sheet.
- The thermal boundary layer thickness shows opposite behavior as the Eckert number E and radiation number K of exponentially stretching sheet.
- In exponential stretching sheet the temperature profile increases with the decline in Prandtl number.
- On the contrary Eckert and radiation number (E and K) have the same effect (increase) with the increase in temperature profile.
- The velocity profile does not get affected by variation of Prandtl number Pr. For Prandtl number the linear, nonlinear and exponential sheets face a decline.
- The thermal boundary layer thickness decreases with increase in Prandtl number Pr. For particular Prandtl number the linear, nonlinear and exponential sheets normally increase.

- In lower branch, the stretching/shrinking parameter λ boost the velocity profile whereas behavior is reverse in upper branch.
- The evaluation of Nusselt number Nu_x, decreases with the increase of Prandtl number Pr.

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