

**Buoyancy assisting and opposing flow of Jeffrey
fluid model with zero mass flux of nanoparticle
along a vertically moving surface**



Thesis Submitted By

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Abstract

The aim of writing this thesis is to express the concept of Buoyancy assisting and opposing flow of Jeffrey fluid model with zero mass flux of nanoparticle along a vertically moving surface in the following ways. The structure of the thesis is developed in the following pattern. In the first chapter, comprehensive study and literature is discussed for Jeffrey fluid model with Buoyancy assisting and opposing flow due to moving surface. Study of nanoparticle's behaviour and their effective thermal performance for various models are reviewed. Second chapter consists of basic definitions of fluids and fundamental laws. Third chapter is a review chapter work for numerical study of Jeffrey nanofluid flow along a stretching sheet. In this chapter momentum, energy and concentration equations are derived by mean of tensor derivation. Boundary layer approach is applied to obtain the simplified form partial differential equations. Later on, by utilizing suitable boundary layer strategy and similarity transformations, the governing systems of PDE's have been converted into a different cumulative ODE's and then solved numerically. Graphically, the related parameters of non-Newtonian nanofluid are solved numerically for local Nusselt number and Sherwood number. In the fourth chapter, a mathematical model is extended for Buoyancy assisting and opposing flow with active/passive control of nanoparticles. The same procedure is adopted to solve the model as discussed for chapter 3. The conclusion of the entire study is presented in chapter 5.

List of contents

1	Introduction	1
2	Fundamental concepts and definitions	4
2.1	Fluid	4
2.2	Fluid mechanics	4
2.3	Physical properties of fluid	4
2.3.1	Density of fluid	4
2.3.2	Dynamic viscosity	4
2.3.3	Fluid's kinematic viscosity	5
2.4	Classification of fluid	5
2.4.1	Inviscid fluid.....	5
2.4.2	Viscous fluid.....	5
2.4.3	Compressible fluid.....	6
2.4.4	Incompressible fluid	6
2.5	Two-Dimensional flow	6
2.6	Boundary layer	6
2.7	Nanofluid Flow	6
2.8	Brownian Motion	6
2.9	Thermophoresis.....	7
2.10	Buoyant force	7
2.11	Some useful non-dimensional numbers	7
2.11.1	Prandtl number	7
2.11.2	Reynolds number	7
2.11.3	Lewis number	7
2.11.4	Nusselt number	7
2.11.5	Sherwood number.....	8
2.12	Conservation laws	8
3	Numerical solution of non-Newtonian nanofluid flow over a stretching sheet	9
3.1	Mathematical Formulation of the problem.....	9
3.2	Methodology	14
3.3	Results and discussion.....	15

4	Buoyancy assisting and opposing flow of Jeffrey fluid model with zero mass flux of nanoparticle along a vertically moving surface.....	24
4.1	Formulation of the problem	24
4.2	Methodology	27
4.3	Results and discussion.....	28
5	Concluding remarks	50
	References	52

List of figures

1. Geometry of the model.....	9
2. Effects of β on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$	17
3. Effects of λ on $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$	17
4. Effects of N_b on $\theta(\eta)$ and $\phi(\eta)$	18
5. Effects N_t on $\theta(\eta)$ and $\phi(\eta)$	18
6. Effects of Le on $\theta(\eta)$ and $\phi(\eta)$	19
7. Variation of Nusselt number with respect to N_b and N_t (taking $Pr=1.0$)	20
8. Variation of Nusselt number with respect to N_b and N_t (taking $Pr=4.0$)	20
9. Variation of Nusselt number with respect to N_b and N_t (taking $Le=1.0$).....	21
10. Variation of Nusselt number with respect to N_b and N_t (taking $Le=8.0$)	21
11. Variation of Sherwood number with respect to N_b and N_t (taking $Pr=1.0$).....	22
12. Variation of Sherwood number with respect to N_b and N_t (taking $Pr=4.0$).....	22
13. Variation of Sherwood number with respect to N_b and N_t (taking $Le=1.0$).....	23
14. Variation of Sherwood number with respect to N_b and N_t (taking $Le=8.0$).....	23
15. Geometry of the model.....	25
16. Effects of β on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$	32
17. In assisting flow effects of β on $\theta(\eta)$	33
18. In opposing flow effects of β on $\theta(\eta)$	33
19. In assisting flow effects of β on $\phi(\eta)$	33
20. In opposing flow effects of β on $\phi(\eta)$	33
21. Effects of λ on $f'(\eta)$	34
22. In assisting flow effects of λ on $\theta(\eta)$	35
23. In opposing flow effects of λ on $\theta(\eta)$	35
24. In assisting flow effects of λ on $\phi(\eta)$	35
25. In opposing flow effects of λ on $\phi(\eta)$	35
26. Effects of N_b on $f'(\eta)$	36
27. In assisting flow effects of N_b on $\theta(\eta)$	37
28. In opposing flow Effects of N_b on $\theta(\eta)$	37
29. In assisting flow effects of N_b on $\phi(\eta)$	37
30. In opposing flow effects of N_b on $\phi(\eta)$	37
31. Effects of N_t on $f'(\eta)$	38
32. In assisting flow effects of N_t on $\theta(\eta)$	39
33. In opposing flow effects of N_t on $\theta(\eta)$	39
34. In assisting flow effects of N_t on $\phi(\eta)$	39
35. In opposing flow effects of N_t on $\phi(\eta)$	39

36. Effects of Pr on $f'(\eta)$	40
37. In assisting flow effects of Pr on $\theta(\eta)$	41
38. In opposing flow effects of Pr on $\theta(\eta)$	41
39. In assisting flow effects of Pr on $\phi(\eta)$	41
40. In opposing flow effects of Pr on $\phi(\eta)$	41
41. Effects of Le on $f'(\eta)$	42
42. In assisting flow effects of Le on $\theta(\eta)$	43
43. In opposing flow effects of Le on $\theta(\eta)$	43
44. In assisting flow effects of Le on $\phi(\eta)$	43
45. In opposing flow effects of Le on $\phi(\eta)$	43
46. Effects of Gr on $f'(\eta)$	44
47. In assisting flow effects of Gr on $\theta(\eta)$	45
48. In opposing flow effects of Gr on $\theta(\eta)$	45
49. In assisting flow effects of Gr on $\phi(\eta)$	45
50. In opposing flow effects of Gr on $\phi(\eta)$	45
51. Effects of R on $f'(\eta)$	46
52. In assisting flow effects of R on $\theta(\eta)$	47
53. In opposing flow effects of R on $\theta(\eta)$	47
54. In assisting flow effects of R on $\phi(\eta)$	47
55. In opposing flow effects of R on $\phi(\eta)$	47
56. In active control of nanoparticles effects of r on $f'(\eta)$	48
57. In passive control of nanoparticles effects of r on $f'(\eta)$	48
58. In assisting flow effects of r on $\theta(\eta)$	49
59. In opposing flow effects of r on $\theta(\eta)$	49
60. In assisting flow effects of r on $\phi(\eta)$	49
61. In opposing flow effects of r on $\phi(\eta)$	49
62. In assisting flow variation of Nusselt number with respect to N_b and N_t for $Pr = 1$	50
63. In opposing flow variation of Nusselt number with respect to N_b and N_t for $Pr = 1$	50
64. In assisting flow variation of Nusselt number with respect to N_b and N_t for $Pr = 4$	50
65. In opposing flow variation of Nusselt number with respect to N_b and N_t for $Pr = 4$	50
66. In assisting flow variation of Nusselt number with respect to N_b and N_t for $Le= 1$	51
67. In opposing flow variation of Nusselt number with respect to N_b and N_t for $Le= 1$	51
68. In assisting flow variation of Nusselt number with respect to N_b and N_t for $Le= 8$	51
69. In opposing flow variation of Nusselt number with respect to N_b and N_t for $Le= 8$	51
70. In assisting flow variation of Sherwood number with respect to N_b and N_t for $Pr = 1$	52
71. In opposing flow variation of Sherwood number with respect to N_b and N_t for $Pr = 1$	52
72. In assisting flow variation of Sherwood number with respect to N_b and N_t for $Pr = 4$	52
73. In opposing flow variation of Sherwood number with respect to N_b and N_t for $Pr = 4$	52

- 74. In assisting flow variation of Sherwood number with respect to N_b and N_t for $Le= 1$ 53
- 75. In opposing flow variation of Sherwood number with respect to N_b and N_t for $Le= 1$ 53
- 76. In assisting flow variation of Sherwood number with respect to N_b and N_t for $Le= 8$ 53
- 77. In opposing flow variation of Sherwood number with respect to N_b and N_t for $Le= 8$ 53