Thermal slip analysis of a Casson fluid in a channel

Submitted by

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A dissertation submitted to the department of computer science, Bahria University, Islamabad as a partial fulfillment of the requirements for the award of the degree of the MS

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Dedicated to

My Respected Parents and teachers

Who always support and guide me to build my career

Acknowledgement

All Praise to ALLAH who is the most beneficent, the merciful, give mankind the ability and knowledge to explore the secrets of the universe.

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Umar Farooq

Bahria University Islamabad,

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Abstract

The objective of this thesis is to carry out thermal slip analysis of a Casson fluid in a channel. The fluid is flowing in a channel under the influence of magnetic field and thermal radiation. The channel walls are slippery enough to analyze the slip effects. The flow is confined to a porous zone. We obtained dimensionless flow equations by introducing non-dimensional parameters. The converted equations are solved analytically. The impact of various parameters on flow field are represented graphically. Variation of thermal and velocity slip parameters are closely observed. With an increase in thermal slip, temperature increases at one wall, while decreases at the other. Velocity at both walls increases by increasing the velocity slip.

NOMENCLATURE

x, y	coordinate axis
u, v	velocity components
K	porous permeability
B_0	magnetic field intensity
g	gravitational acceleration
T_0	temperature at lower wall
T'	temperature at upper wall
P	Pressure
K	thermal conductivity
Re	Reynolds Number
S	thermal conductivity parameter
H	Hartmann's Number
Pe	Peclet Number
N	thermal Radiation Parameter
Gr	Grashof Number

Greek Letters

μ	dynamic viscosity
ν	kinematic viscosity
ρ	density of fluid
σ_e	electrical conductivity
β	volumetric Expansion
β_1	Casson Parameter
α	thermal Radiation
ϕ_1	velocity Slip parameter at lower wall
ϕ_2	velocity Slip parameter at upper wall

γ	cold wall Slip Parameter
σ	heated Wall Slip Parameter
γ_1	thermal Slip parameter at lower wall
γ_2	thermal Slip parameter at Upper wall

List of Abbreviations

C F	Casson fluid
N N	Non-newtonian
N F	Nano fluid
P C	Porous channel

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Chapter 1

Introduction

This chapter focuses on basic definitions and equations pertaining to fluid flowing through a channel. It also focuses, on explaining the solution method to the flow problem under discussion.

1.1 Definitions and concepts

1.1.1 Fluid

A fluid is any substance that continuously deform under the action of shear stress, however small the shear stress is. Fluids are distinguished as, ideal and real fluids. Its daily life examples are water, honey, and blood etc.

1.1.2 Real fluid

Real fluids possess certain viscosity, they are categorized as Newtonian and Non-Newtonian fluids.

1.1.3 Ideal Fluid

Ideal fluids are those fluids that posses negligible viscosity.

1.1.4 Fluid mechanics

Fluid mechanics is the branch of engineering that illustrates the behavior of fluids at rest or in motion. It has immense applications in different areas including engineering and technology, astrophysics, blood flow analysis, and many other fields.

1.1.5 Non- Newtonian Fluids

Non-Newtonian fluids do not obey Newton's law of viscosity. Honey, shampoo, and tooth paste are some of the examples. The Non-Newtonian fluids have numerous applications in chemical industries, food and beverages, biological sciences, and many other areas.

1.1.6 Casson fluid

A Casson fluid has infinite viscosity at zero stress, a stress below which no flow can occur and zero viscosity at infinite stress. Its daily life examples are tomato ketchup, soup, jelly and human blood.

1.1.7 Channel flow

Channel flow is a flow of liquid in a conduit. Fluid flowing through a channel has vast applications in solar equipment, nuclear reactors and civil engineering.

1.1.8 Heat transfer

Transfer of heat is associated with the change in temperature within a channel. This phenomena has immense significance in different areas including medical equipment, heating and cooling systems, power generation and chemical processing. Heat can be transferred through conduction, convection and radiation.

1.1.9 Magnetohydrodynamics

It discusses impact of magnetic field on fluid flowing through a channel. It has many applications in medical sciences, petroleum industry, nuclear technology and engineering.

1.1.10 Thermal slip

Due to thermal slip, shear stress at the wall due to slip, should be included to measure the heat flux.

1.2 Basic laws

The basic equations representing the flow problem are given below.

1.2.1 Continuity Equation

For compressible fluid, the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0. \tag{1.1}$$

1.2.2 Momentum Equation

The momentum equation is given as

$$\rho \frac{dV}{dt} = -\nabla p + \mu \nabla^2 V + \rho g. \quad (1.2)$$

1.2.3 Energy Equation

The energy equation is given as

$$\rho C_p \frac{dT}{dt} = k\nabla^2 T + \mu (\frac{\partial u}{\partial y})^2. \quad (1.3)$$

1.3 Solution method

Ordinary and Partial Differential equations are commonly used to model the problem under consideration. The equations may be linear or non-linear. We calculate dimensionless flow equations by introducing non-dimensional parameters. Then we solve these equations by any appropriate method. The most common methods employed are as follows.

1.3.1 Analytical Methods

The analytical methods include the following methods:

- Perturbation technique
- Homotopy analysis
- Method of separation of variables

1.3.2 Method of separation of variables

The method used to solve the flow problem in our case is illustrated by the following example. The differential equation used in our example is non-homogeneous differential equation of order 3. First, we calculate the characteristic equation as explained in example.

Example

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 4y = e^{2x}Cos3x$$

The characteristic equation is

$$\mathrm{D}^3 + D^2 - 4D - 4 = 0$$

The roots of the equation are $D = 2, -2, -1$

Therefore, the complementary function is

$$y_c = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{2x}$$

The Particular Integral is given by

$$y_p = \frac{1}{(D-2)(D+2)(D+1)} e^{2x} cos3x$$

$$= e^{2x} \frac{1}{D(D+4)(D+3)} \; cos3x$$

$$= e^{2x} \frac{Cos3x}{(D^3 + 7D^2 + 12D)}$$

$$= \; e^{2x} \frac{Cos3x}{-9D - 63 + 12D}$$

$$= e^{2x} \frac{(D+21)\; Cos3x}{3(D^2 - 441)}$$

$$= e^{2x} \frac{-1}{(3)(450)} (-3sin3x + 21cos3x)$$

$$= \frac{e^{2x}}{450} (sin3x - 7cos3x).$$

Chapter 2

Literature Review

Several studies have been conducted related to flow of a CF in a channel. A CF is a NN fluid having infinite viscosity at zero stress, a stress below which no flow can occur and zero viscosity at infinite stress. It has numerous applications in different technologies and biological flows including respiratory system, circulatory system and blood flows. Casson [1] studied flow equations for the pigments oils suspensions. Kunnegowda et al. [2] discusses convective flow of CF in a micro-channel. Makinde et al. [3] discussed peristaltic flow of CF in a channel. Usman et al. [4] discussed Casson NN flow on inclined cylinder. Eegunjobi et al. [5] discussed flow of radiating CF having mixed convection. Hayat et al. [6] discussed flow of CF having nano particles. Tripathi et al. [7] studied flow of CF due to metachronal wave propulsion. Tasawar et al. [8] explained squeezing flow of a CF. Sarojamma et al. [9] analyzed CF flow in a vertical channel with stretched walls.

MHD flow has gained the attention of many researchers due to its immense applications in different fields including medical sciences, petroleum industry, nuclear technology and engineering. The flow of fluid under the influence of magnetic field is called MHD (magneto hydrodynamics) flow. Several researchers have worked in this area. Akbar et al. [10] discussed MHD slip flow in an asymmetric channel. He actually discusses peristaltic flow having carbon nanotubes and explained the behavior of flow under MHD. Saqib et al. [11] analyze blood flow treating it as CF under MHD. The fluid is flowing in horizontal cylindrical tube, the study reveals that the velocity of blood decreases under MHD. Saqib et al. [12] numerically investigated MHD blood flow in a stenosed arteries. The study reveals that increasing magnetic field up to 8 T doesn’t harm the arterial wall. Jawad et al. [13] discusses many solutions of MHD CF flow in a channel. Raza et al. [14] explained MHD NF flow in a rotating channel. The study reveals that fluid flow is controlled by changing the magnetic field. Das et al. [15] analyzed entropy generation in MHD flow. The study shows that entropy increases under MHD.

Many researchers have worked on fluid flowing through porous channel, since it has many applications in medical technology including dialysis of blood in artificial kidney and flow in oxygenators. It has also many engineering applications including design of filters and gaseous diffusion problems. A. Sinha et al. [16] discussed MHD flow of a third order flow in a PC, it has important application in blood flow in a cardiovascular system when magnetic field is applied across. Adesanya et al. [17] studied a flow of a reactive viscous fluid in a PC. The study reveals that porous medium with low permeability depletes the energy in a thermo fluid. Suripeddi Srinivas et al. [18] analyzed Pulsating flow of CF in a PC. The study reveals that by increasing Casson parameter the velocity increases.

The slip flow has many applications, so many researchers have worked in this area. Shashikumar et al. [19] focused on slip flow of CF in a micro channel. The study reveals that entropy increases by increasing radiation parameter. Tauseef et al. [20] worked on flow of NF having thermal slip in converging and diverging Channels. Hayat et al. [21] explains slip flow of NF in a channel. The study shows that velocity slip parameter decelerates the dimensionless velocity, but this is more prominent in the upper part of channel. Hong et al. [22] studied thermal slip boundary conditions. This study explains the reason of inclusion of shear work term. lbanez et al. [23] discussed NF thermal slip flow in a PC. Akbar et al. [24] discussed peristaltic slip flow of Williamson fluid. Hayat et al. [25] discusses impact of slip on peristaltic flow. Shojaeian et al. [26] discussed heat transfer in Newtonian and NN fluid with slip. The results indicate that increase in slip parameters decreases the entropy generation rate. Abbaszadeh et al. [27] explained NF flow having slip in a micro channel. The study depicts that increase in H increases the entropy. Eegunjobi et al. [28] analyzed impact of slip on entropy in a PC. Ibrahim et al. [29] discussed Casson NF flow having slip on stretching surface. Shashikumar et al. [30] explains the impact of second order slip on Casson NF flow. Uddin et al. [31] studied flow of radiative convective NF slip flow on a shrinking sheet. Ranjit et al. [32] studied micro-channel slip flow with different zeta potential. Malvandi et al. [33] studied thermophoresis on slip flow of NF. Raisi et al. [34] numerically investigated NF flow in a micro channel with both slip and no slip. Torabi et al. [35] discussed entropy in micro PC with slip. Shaw et al. [36] describes the NF flow on a stretched surface. Cheema et al. [37] numerically investigated squeezing flow with slip. This study highlights that by increasing temperature slip, the temperature of fluid at lower disc increases but reverse behavior is seen at upper disc. None of these studies focuses on thermal slip analysis of a CF.

The focus of our study is on thermal slip analysis of a Casson fluid in a channel.

Chapter 3

Flow of viscous fluid through a porous zone in a channel with slippery walls

In this chapter we studied, MHD Oscillatory slip flow in a channel filled with porous medium with time dependent boundary conditions. The flow equations are non-dimensionalised and resolved into harmonic and non-harmonic parts. We calculate exact solution for temperature and velocity field and understand impact of various parameters on flow variables. This chapter is a review of paper by Makinde et al. [38]

3.1 Mathematical analysis

We consider two dimensional viscous, incompressible, and electrically conducting fluid flowing in a channel with non-uniform wall temperature. The fluid is under the impact of magnetic field and thermal radiation. The channel walls are located at y=0 and y=a and are slippery enough to analyze the slip effects. We choose a cartesian coordinate system presented in figure below.

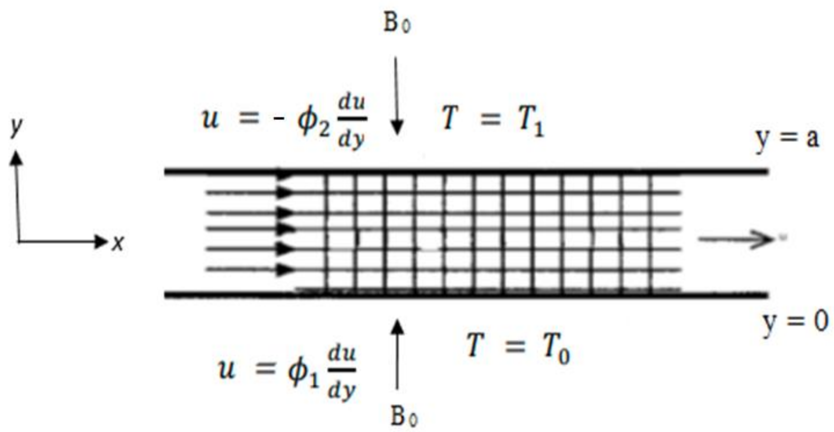

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Fig. 3.1: Flow model of problem

The velocity and temperature field are given as

$$V = (u'(y,t), 0, 0) \,, \tag{3.1}$$

$$T = T(y,t). \tag{3.2}$$

In vector form, the flow equations are given as

$$\nabla \cdot \mathrm{V} = 0, \tag{3.3}$$

$$\frac{dV}{dt} = -\frac{1}{\rho}\nabla p + \nu\, \nabla^2 V + \frac{1}{\rho}\ (J \times B), \tag{3.4}$$

$$\rho C_p \frac{dT}{dt} = k\nabla^2 T + \mu(\frac{\partial u}{\partial y})^2. \tag{3.5}$$

Using the velocity and temperature field (3.1), (3.2) to (3.3) to (3.5), the flow governing equations for the problem are given as

$$\frac{\partial u'}{\partial t'} = \frac{-1}{\rho}\ \frac{dP'}{dx'} + \nu\ \frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K}u' - \frac{\sigma_e {B_0}^2}{\rho}u' + g\beta(T' - T_0), \tag{3.6}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_f}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho c_p}(T' - T_0). \tag{3.7}$$

The boundary conditions are

$$u' = \phi_1 \frac{du'}{dy'} \quad , \quad T' = T_0 \quad \text{at} \quad y' = 0, \tag{3.8}$$

$$u' = -\phi_2 \frac{du'}{dy'} \quad , \quad T' = T_1 \quad \text{at} \quad y' = a. \tag{3.9}$$

We introduced the dimensionless parameters given as

$$(x, y) = \frac{(x', y')}{h}\ ,\quad u = \frac{hu'}{\nu},\ P = \frac{h^2 P'}{\rho \nu^2},\ \ Gr = \frac{g\beta(T_1 - T_0)h^3}{\nu^2},$$

$$Pr = \frac{\rho c_p \nu}{\mathrm{k}}, \gamma = \frac{\phi_1}{h}, \theta = \frac{T' - T_0}{T_1 - T_0},\ \delta = \frac{4\alpha^2 h^2}{\rho c_p \nu}, \sigma = \frac{\phi_2}{h}\ ,$$

$$H^2 = \frac{\sigma_e {B_0}^2 h^2}{\rho\nu}, Da = \frac{K}{h^2}, s^2 = \frac{1}{Da}, \lambda = -\frac{dP}{dx} \tag{3.10}$$

The dimensionless flow equations are

$$Re\frac{\partial u}{\partial t} = \lambda + \frac{\partial^2 \mathrm{u}}{\partial y^2} - (H^2 + S^2)u + Gr\ \theta, \tag{3.11}$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta. \tag{3.12}$$

The boundary conditions are

$$u = \gamma\frac{du}{dy}, \quad \theta = 0 \quad \text{at} \quad y = 0, \tag{3.13}$$

$$u = -\sigma\frac{du}{dy}, \quad \theta = 1 \quad \text{at} \quad y = 1. \tag{3.14}$$

3.2 Analytical Procedure

Let us consider solution for pulsatile flow is of the form

$$\lambda = \lambda_0 + \lambda_1 e^{i\omega t}, u(t,y) = R(y) + G(y)e^{i\omega t}, \theta(t,y) = P(y) + Q(y)e^{\ i\omega t}. \qquad (3.15)$$

Putting values from Eq. (3.15) to Eq. (3.11) – (3.14), we have the following differential equations

$$R''\,(\mathrm{y}) - (H^2 + s^2)R(y) = -GrP(y) - \lambda_{0,}, \qquad (3.16)$$

$$G''\,(\mathrm{y}) - (H^2 + s^2 + Re\ i\omega)G(y) = -GrQ\ (y) - \lambda_1, \qquad (3.17)$$

$$P''\,(\mathrm{y}) + N^2 P(y) = 0, \qquad (3.18)$$

$$Q''\,(\mathrm{y}) + (N^2 - \ i\omega Pe)Q(y) = 0. \qquad (3.19)$$

With the boundary conditions

$$R(0) = \gamma R'(0), \qquad R(1) = -\sigma R'(1), \qquad (3.20)$$

$$G(0) = \gamma G'(0), \qquad G(1) = -\sigma G'(1), \qquad (3.21)$$

$$P(0) = 0, \qquad P(1) = 1\,, \qquad (3.22)$$

$$Q(0) = 0, \qquad Q(1) = 0. \qquad (3.23)$$

On solving Eq. (3.16)-(3.19) with the boundary conditions (3.20)-(3.23), we have

$$P(y) = \frac{Sin(Ny)}{\mathrm{Sin}(N)}\,, \qquad (3.24)$$

$$Q(y) = 0\,, \qquad (3.25)$$

$$R(y) = C_1 e^{m_2 y} + C_2 e^{-m_2 y} + J_2 + J_1 Sin(Ny), \qquad (3.26)$$

$$G(y) = C_3 e^{m_3 y} + C_4 e^{-m_3 y} + J_9. \qquad (3.27)$$

Where the values of constants are given in appendix 1.

The rate of heat transfer at channel wall is given as

$$Nu = -\frac{\partial\theta}{\partial y} = -\frac{Ncos(Ny)}{\sin(N)}. \qquad (3.28)$$

While shear stress is given as

$$\tau = -\mu \frac{\partial u}{\partial y} = - \ A'(y) - B'(y)e^{i\omega t}. \tag{3.29}$$

3.3 Results and Discussion

We have calculated exact solution of equation (3.16) to (3.19) with boundary conditions (3.20) to (3.23). Figure 3.1 indicates that velocity increases by increasing cold wall parameter. Figure 3.2 indicates that, by increasing H velocity decreases which is result of Lorentz force at heated wall. Figure 3.3 shows that increase in S decrease velocity. Figure 3.4 indicates that velocity increases by increasing σ. Moreover, by increasing heated wall slip, decreases velocity to minimal at cold wall and more at heated wall which causes flow reversal.

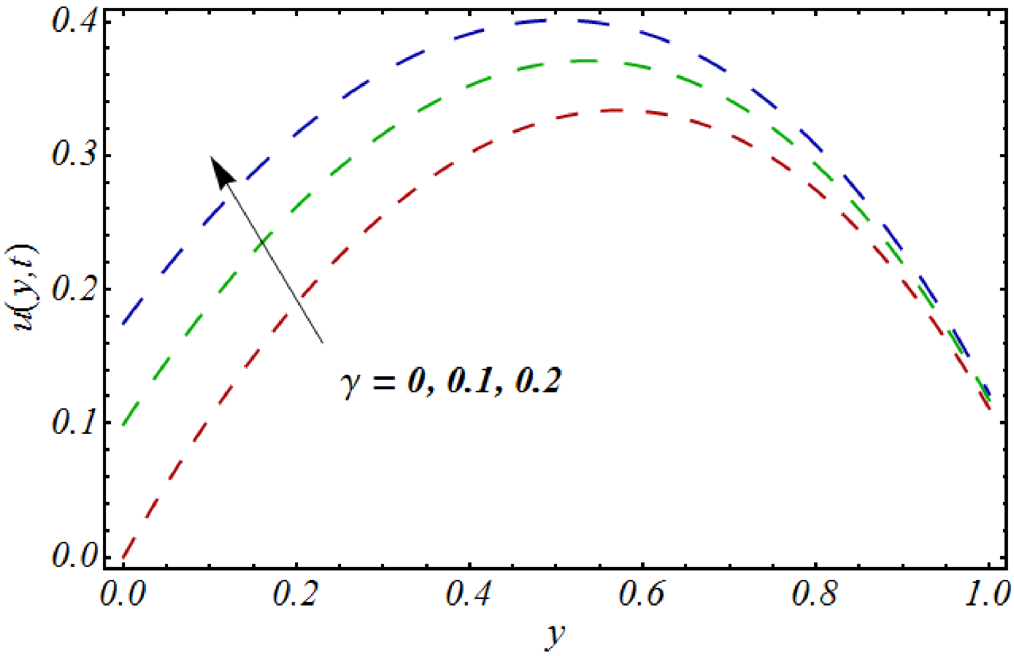


Fig. 3.2: $u(y,t)$ with γ.

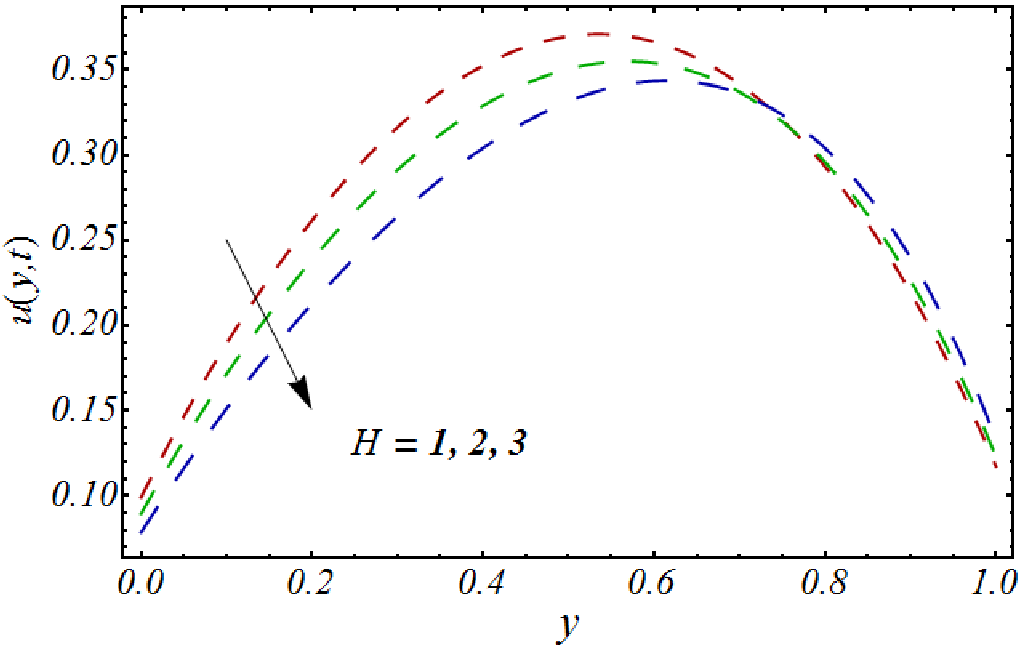


Fig. 3.3: $u(y,t)$ with H.

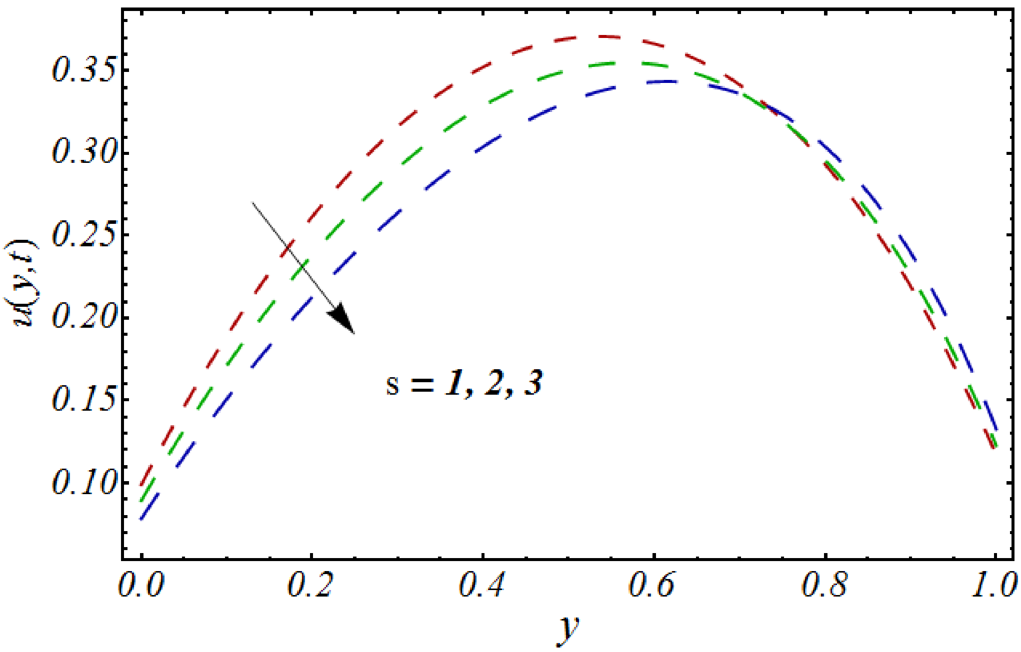


Fig. 3.4: $u(y,t)$ with s.

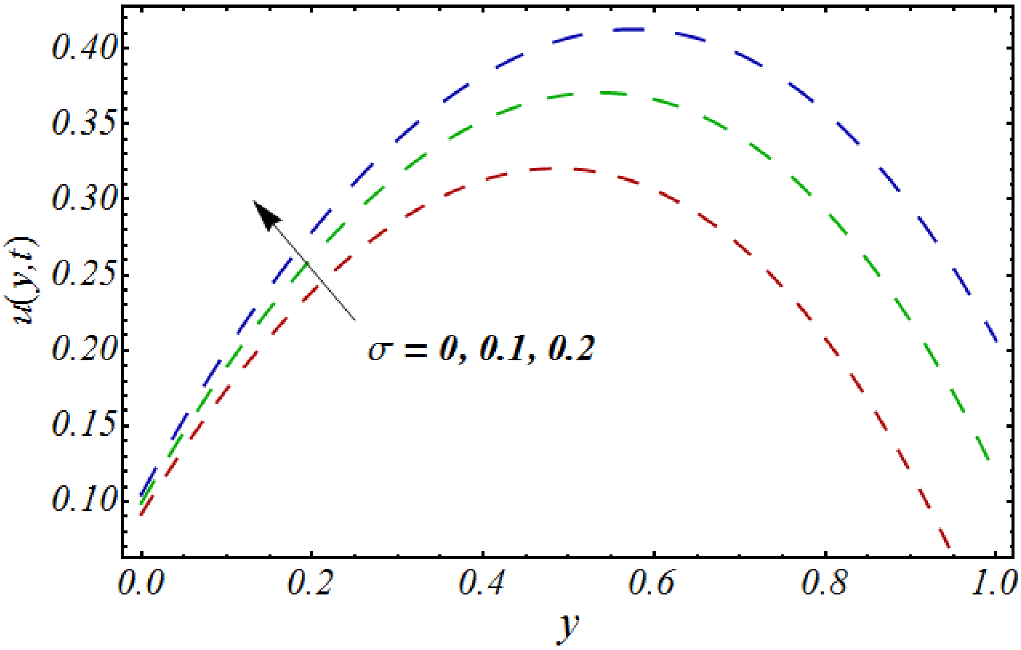


Fig. 3.5: $u(y,t)$with σ.

Chapter 4

Thermal slip analysis of a Casson fluid in a channel

In this section we examined the flow of a Casson fluid in a channel with velocity and thermal slips at the walls. The flow is under the impact of magnetic field. The same technique as in previous chapter is employed to calculate the solution of the problem. The impact of various parameter along with thermal slip on flow variables are discussed. Section 4.1 is devoted to problem formulation, Section 4.2 is devoted to analytical solution, while in section 4.3 we have results and discussions.

4.1 Problem formulation

We consider two dimensional viscous, incompressible, and electrically conducting Casson fluid flowing in a channel. The fluid is under the impact of magnetic field and thermal radiation. The channel walls are at y=0 and y=a and are slippery enough to analyze the slip effects i.e. the velocity and thermal slip. We choose a Cartesian coordinate system presented in figure below.

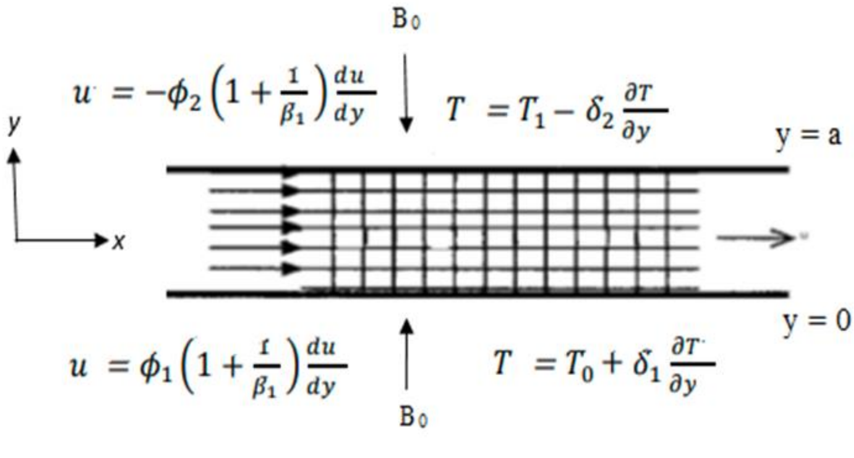


Fig. 4.1: Flow model of problem

The velocity and temperature field are given as

$$V = (u'(y,t), 0, 0)\ , \tag{4.1}$$

$$T = T(y,t). \tag{4.2}$$

The constitutive equation for Casson fluid is given as

$$\tau_{ij} = \begin{cases} \frac{2(\mu_B + P_y)}{2\pi} e_{ij}\ , & \pi > \pi_c \\ \frac{2(\mu_B + P_y)}{\sqrt{2\pi_c}} e_{ij}\ , & \pi < \pi_c \end{cases} \tag{4.3}$$

where τ_{ij} is $(i,j)th$ component of stress tensor, e_{ij} is (i,j) component of deformation, $\pi = e_{ij}\ e_{ij}$, π_c is critical value of product for non-Newtonian model, μ_B is plastic dynamic viscosity and P_y is yield stress.

The flow governing equations for the problem are given:

$$\frac{\partial u'}{\partial t'} = \frac{-1}{\rho}\frac{dp'}{dx'} + \left(1 + \frac{1}{\beta_1}\right)\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K}u' - \frac{\sigma_e {B_0}^2}{\rho}u' + g\beta(T' - T_0), \tag{4.4}$$

$$\frac{\partial T'}{\partial t'} = \frac{k_f}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho c_p}(T' - T_0). \tag{4.5}$$

where β_1 is a the Casson parameter.

The boundary conditions are

$$u' = \phi_1\left(1 + \frac{1}{\beta_1}\right)\frac{du'}{dy'} \quad \text{at} \quad y' = 0, \tag{4.6}$$

$$u' = -\phi_2\left(1 + \frac{1}{\beta_1}\right)\frac{du'}{dy'} \quad \text{at} \quad y' = a, \tag{4.7}$$

$$T' = T_0 + \delta_1\frac{\partial T'}{\partial y'} \quad \text{at} \quad y' = 0, \tag{4.8}$$

$$T' = T_1 - \delta_2\frac{\partial T'}{\partial y'} \quad \text{at} \quad y' = a. \tag{4.9}$$

where δ_1 and δ_2 are thermal slip parameters.

We introduced the dimensionless parameters given below:

$$(x, y) = \frac{(x', y')}{h}\ , \quad u = \frac{hu'}{\nu},\ P = \frac{h^2 P'}{\rho\nu^2},\ Gr = \frac{g\beta(T_1 - T_0)h^3}{\nu^2},\ Pr = \frac{\rho c_p \nu}{\mathrm{k}},$$

$$\gamma = \frac{\phi_1}{h}, \theta = \frac{T' - T_0}{T_1 - T_0},\ \delta = \frac{4\alpha^2 h^2}{\rho c_p \nu}, \sigma = \frac{\phi_2}{h}\ , H^2 = \frac{\sigma_e {B_0}^2 h^2}{\rho\nu}, Da = \frac{K}{h^2},$$

$$s^2 = \frac{1}{Da}, \lambda = -\frac{dP}{dx}, \gamma_1 = \frac{\delta_1}{h}\ ,\ \gamma_2 = \frac{\delta_2}{h}. \tag{4.10}$$

The dimensionless parameters given in equation 4.10 are used to obtain dimensionless form of flow equations as

$$Re\frac{\partial u}{\partial t} = \lambda + \left(1+\frac{1}{\beta_1}\right)\frac{\partial^2 u}{\partial y^2} - (H^2+S^2)u + Gr\,\theta, \tag{4.11}$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta. \tag{4.12}$$

The boundary conditions are

$$u = \gamma\left(1+\frac{1}{\beta_1}\right)\frac{du}{dy} \qquad \text{at} \qquad y = 0, \tag{4.13}$$

$$u = -\sigma\left(1+\frac{1}{\beta_1}\right)\frac{du}{dy} \qquad \text{at} \qquad y = 1, \tag{4.14}$$

$$\theta = \gamma_1\frac{\partial \theta}{\partial y} \qquad \text{at} \qquad y = 0, \tag{4.15}$$

$$\theta = 1 - \gamma_2\frac{\partial \theta}{\partial y} \qquad \text{at} \qquad y = 1. \tag{4.16}$$

4.2 Analytical Procedure

Let us consider solution for the problem is of the form

$$\lambda = \lambda_0 + \lambda_1 e^{i\omega t},$$

$$u(t,y) = R(y) + G(y)e^{i\omega t},$$

$$\theta(t,y) = P(y) + Q(y)e^{\ i\omega t}. \tag{4.17}$$

Using Eq. (4.17) in Eq. (4.11) – (4.16), we obtained the following equations:

$$R''(\mathrm{y})\left(1+\frac{1}{\beta_1}\right) - (H^2+s^2)R(y) = -GrP(y) - \lambda_0, \tag{4.18}$$

$$G''(\mathrm{y})\left(1+\frac{1}{\beta_1}\right) - (H^2+s^2+Re\,i\omega)G(y) = -GrQ\,(y) - \lambda_1, \tag{4.19}$$

$$P''(\mathrm{y}) + N^2P(y) = 0, \tag{4.20}$$

$$Q''(\mathrm{y}) + (N^2 - \ i\omega Pe)Q(y) = 0. \tag{4.21}$$

with the boundary conditions

$$R(0) = \gamma\left(1+\frac{1}{\beta_1}\right)R'(0) \qquad ,\ R(1) = -\sigma\left(1+\frac{1}{\beta_1}\right)R'(1), \tag{4.22}$$

$$G(0) = \gamma\left(1 + \frac{1}{\beta_1}\right)G'(0)\,, \qquad G(1) = -\sigma\left(1 + \frac{1}{\beta_1}\right)G'(1), \tag{4.23}$$

$$P(0) = \gamma_1 P'(0), \qquad P(1) = 1 - \gamma_2 P'(1), \tag{4.24}$$

$$Q(0) = \gamma_1 Q'(0), \qquad Q(1) = -\gamma_2 Q'(1). \tag{4.25}$$

On solving Eq. (4.18)-(4.21) with boundary conditions (4.22)-(4.25), we get

$$R(y) = \frac{\gamma_1 N Cos(Ny) + Sin(Ny)}{(\gamma_1+\gamma_2)N\mathrm{Cos}(N) + (1-\gamma_1\gamma_2 N^2)\mathrm{Sin}(N)}, \tag{4.26}$$

$$G(y) = 0, \tag{4.27}$$

$$P(y) = C_1 e^{m_1 y} + C_2 e^{-m_1 y} + n_1 + n_2 Cos(Ny) + n_3 Sin(Ny), \tag{4.28}$$

$$Q(y) = C_3 e^{m_3 y} + C_4 e^{-m_3 y} + n_4. \tag{4.29}$$

where the values of constants are given in appendix 2.

The rate of heat transfer at channel wall is given as

$$Nu = -\frac{\partial \theta}{\partial y} = \frac{-\gamma_1 N^2 sin(Ny) + N cos(Ny)}{(\gamma_1+\gamma_2)N\mathrm{Cos}(N) + (1-\gamma_1\gamma_2 N^2)\mathrm{Sin}(N)}. \tag{4.30}$$

While shear stress is given as

$$\tau = -\mu\frac{\partial u}{\partial y} = -\ A'(y) - B'(y)e^{i\omega t}. \tag{4.31}$$

4.3 Results and Discussion

We have calculated exact solution of equation (4.17) to (4.20) with boundary conditions (4.21) to (4.24). Figure 4.1 indicates that velocity increases with an increase in Casson parameter. However an increase in β_1 indefinitely reduces to review problem. Figure 4.2 indicates that velocity increases by increasing Grashof number. An increase in Gr means increase in temperature gradient which leads to velocity increase. Figure 4.3 and 4.7 indicates that velocity increases by increasing slip parameters γ and σ. It is because of the reason that friction force near the wall decreases, due to slip and hence resistance to the fluid motion decreases and finally velocity increases. Figure 4.4 shows that increase in H decreases velocity which is a result of resistive Lorentz force due to effect of magnetic field on electrically conducting Casson fluid flowing in a channel. Figure 4.5 shows that increase in s decreases velocity, because an increase in Darcy number increases porous permeability and hence velocity decreases. Figure 4.7 shows that increase in γ_1 increases the temperature. Figure 4.8 shows that increase in γ_2 decreases the temperature.

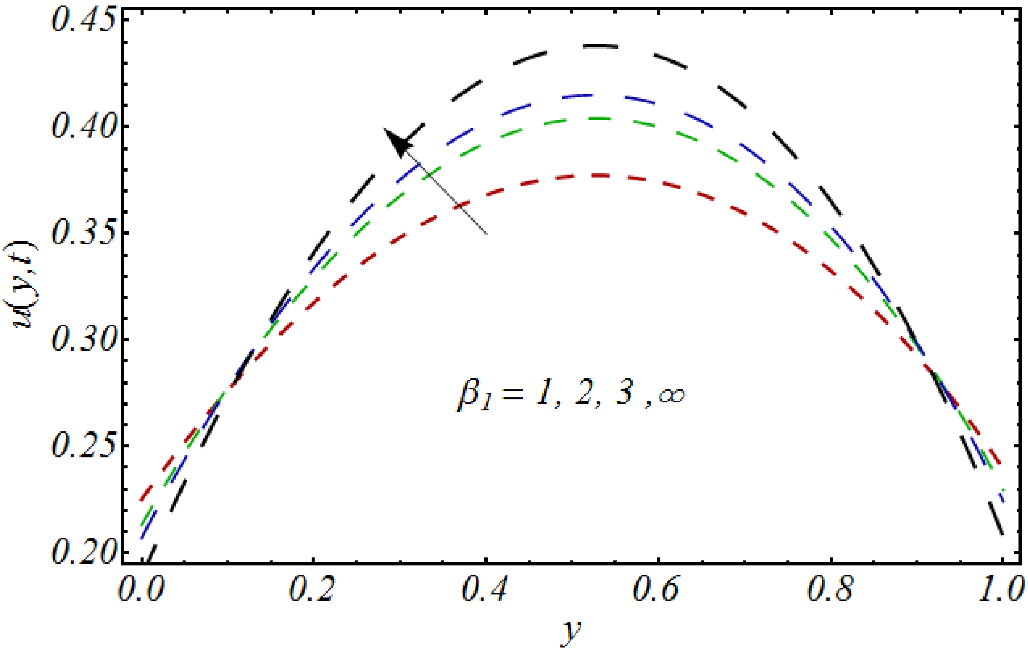


Fig. 4.2: $u(y,t)$ with β_1

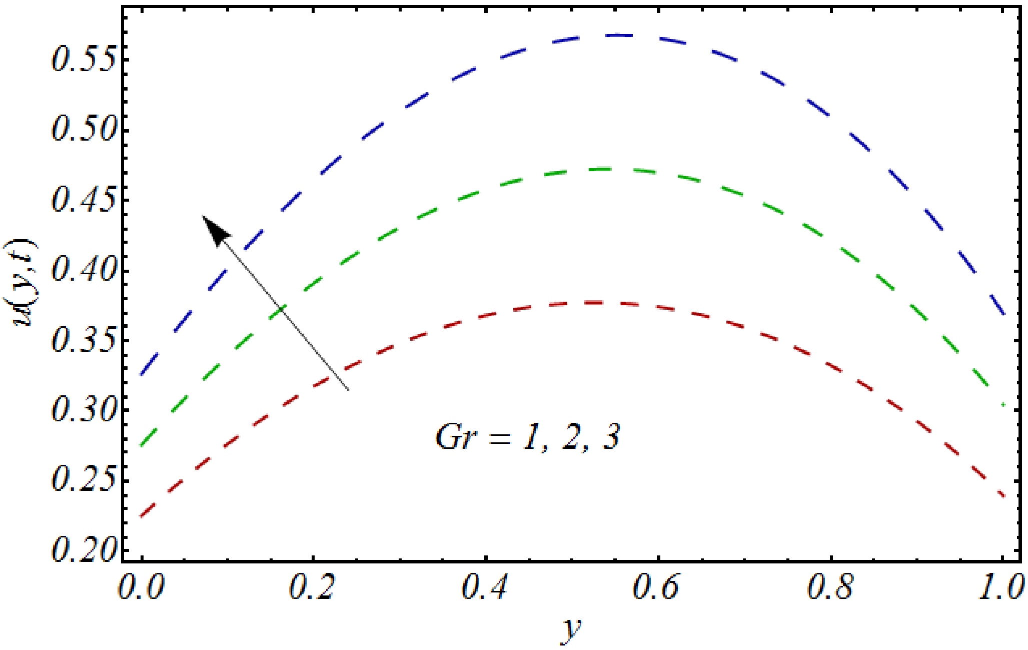


Fig. 4.3: $u(y,t)$ with Gr.

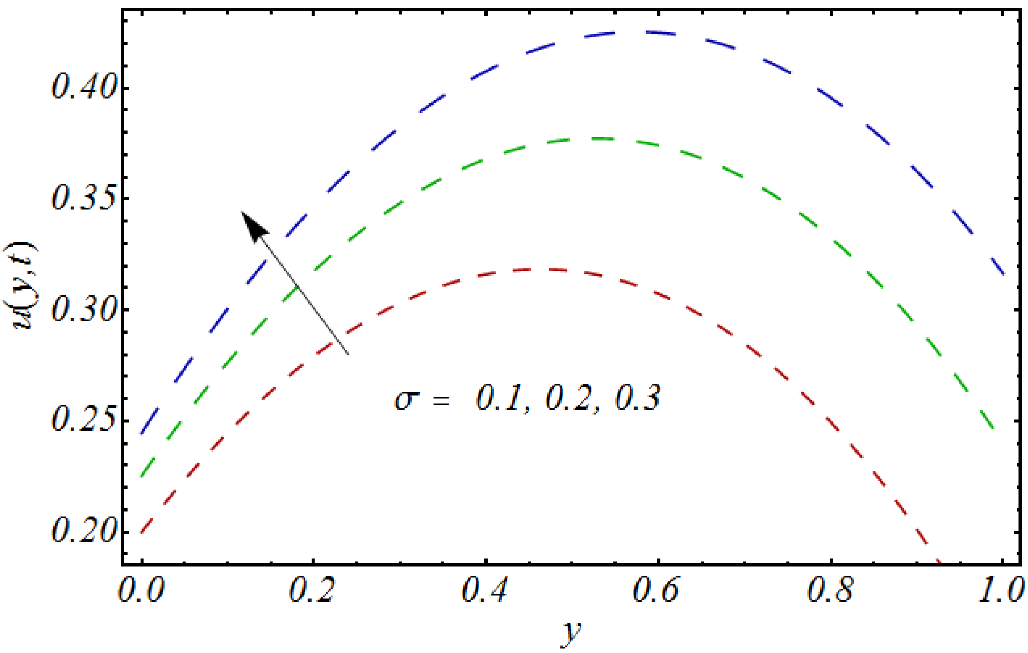
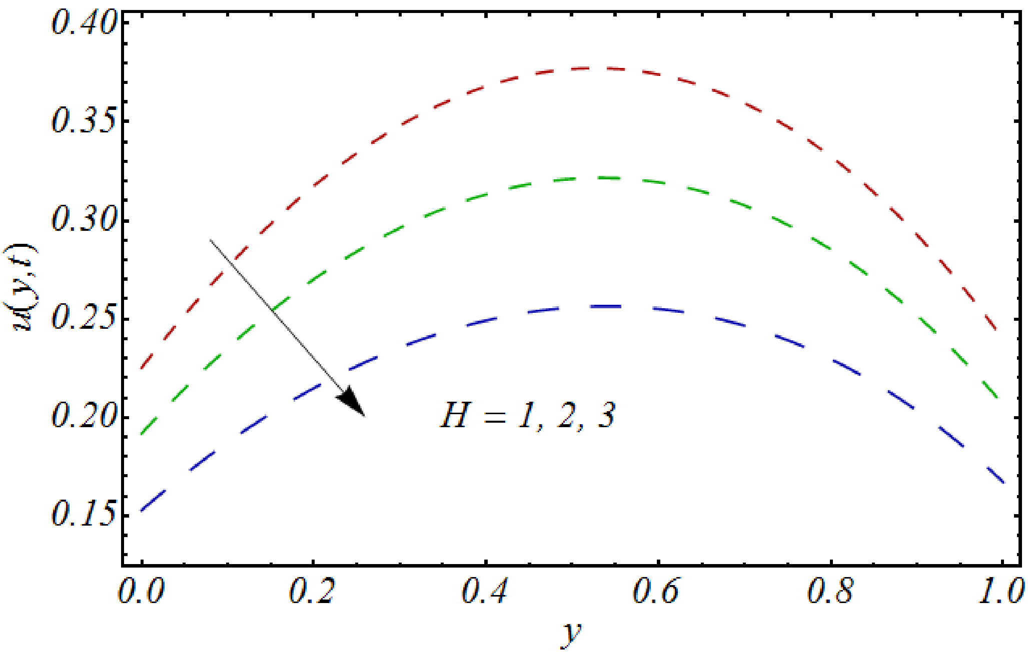


Fig. 4.4: $u(y,t)$ with σ.



.Fig. 4.5: $u(y,t)$ with H.

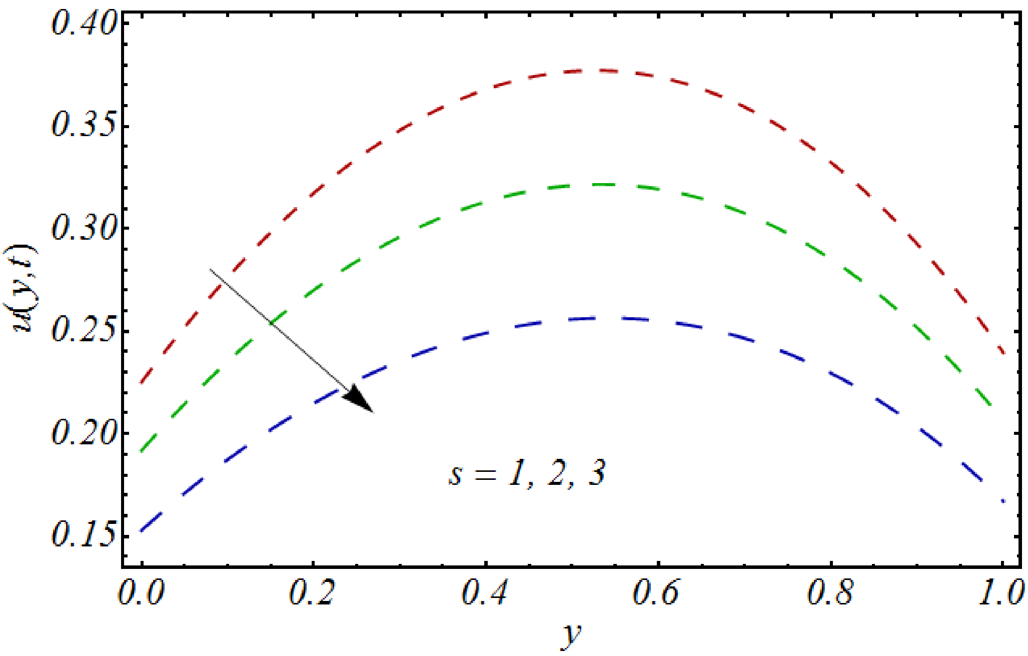


Fig. 4.6: $u(y,t)$ with s.

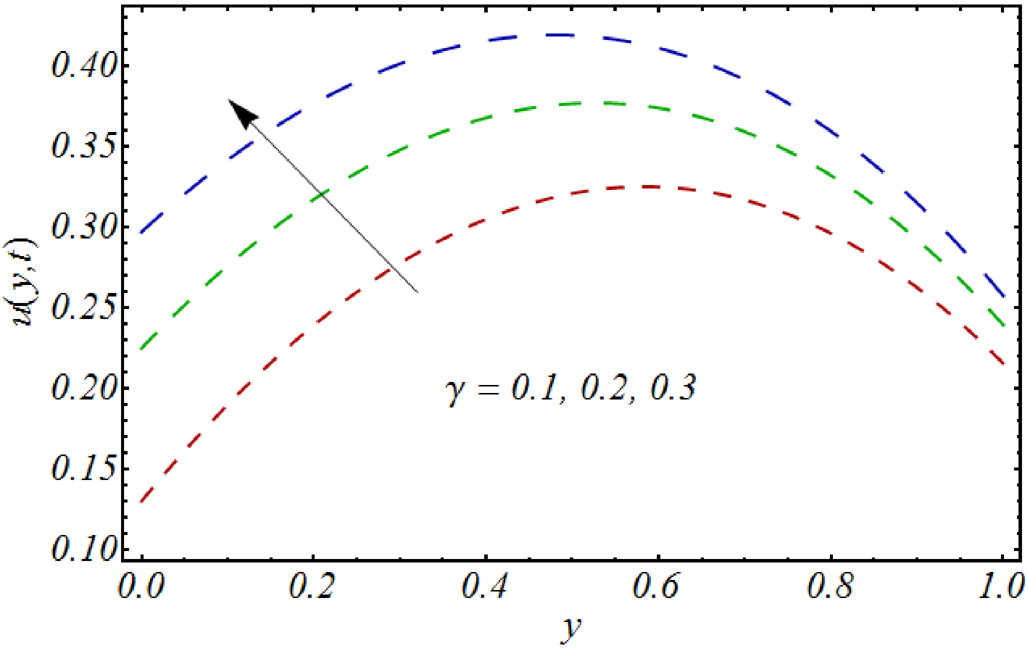


Fig. 4.7: $u(y,t)$ with γ.

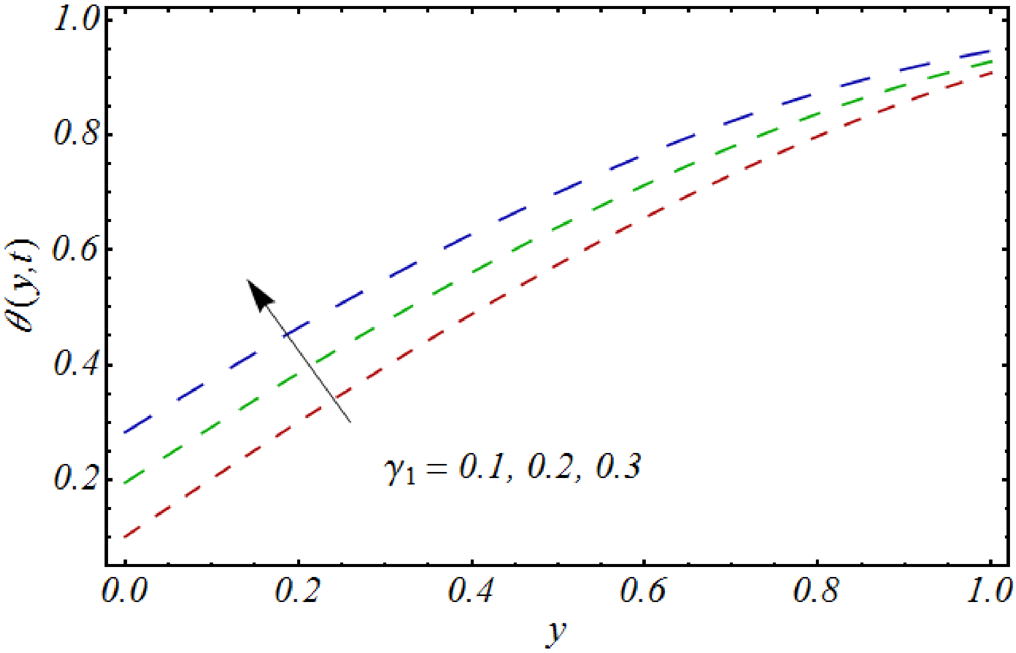


Fig. 4.8: $\theta(y,t)$ with γ_1.

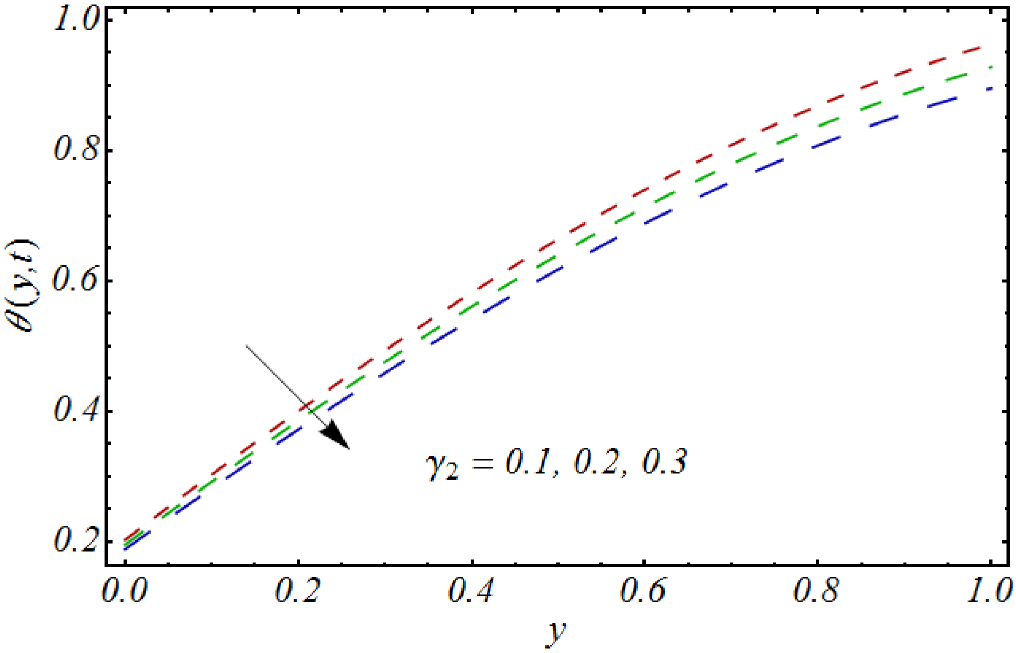


Fig. 4.9: $\theta(y,t)$ with γ_2.

Chapter 5

Conclusions

5.1 Chapter 3

In this chapter we discussed MHD Oscillatory slip flow and heat transfer in a channel filled with porous media. We have the following conclusions.

- The velocity increases by increasing cold wall parameter.
- By increasing Hartmann's number velocity decreases which is result of Lorentz force resistance. However, velocity increases at heated wall due to flow reversal because of the wall slip
- With an increase in Darcy number, which increases porous permeability, fluid velocity decreases.
- With an increase in σ (i.e. heated wall parameter) velocity decrease at cold wall while more decrease is seen at heated wall which causes flow reversal.

5.2 Chapter 4

In this chapter we worked on thermal slip analysis of a Casson fluid in a channel. We have drawn the following conclusions.

- The velocity increases by increasing Casson parameter.
- The velocity increases with an increase in Grashof number.
- The velocity increases by increasing heated wall parameter.
- An increase in Hartmann's number decreases velocity.
- With an increase in Darcy number, which increases porous permeability fluid velocity decreases.
- The velocity increases by increasing cold wall parameter.
- An increase in γ_1 increases the temperature while it decreases with γ_2.

5.3 Comparison of results in chapter 3 and 4

Parameter (increasing value)	**Chapter 3**	**Chapter 4**
$\boldsymbol{\gamma}$ (Cold wall parameter)	Velocity increases	Velocity increases
$\boldsymbol{\sigma}$ (Heated wall parameter)	Velocity decreases	Velocity increases
H **(**Hartmann's number)	Velocity decreases	Velocity decreases
S (Darcy Number)	Velocity decreases	Velocity decreases
$\boldsymbol{\beta_1}$ (Casson parameter)		Velocity increases, increasing β_1 indefinitely reduces to review problem i.e. chapter 3
Gr (Grashof Number)	Not applicable	Velocity increases
$\boldsymbol{\gamma_1}$ (Thermal slip at lower wall)	Not applicable	Temperature increases
$\boldsymbol{\gamma_2}$ (Thermal slip at lower wall)	Not applicable	Temperature decreases

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Appendix 1

$m_1{}^2 = N^2 - \ Pe\, i\omega$, $\quad m_2{}^2 = H^2 + s^2$, $m_3{}^2 = H^2 + s^2 + Re\, i\omega$,

$C_1 = \frac{J_5 J_7 - J_4 J_8}{J_4 J_6 - J_3 J_7}$, $C_2 = \frac{J_3 J_8 - J_5 J_6}{J_4 J_6 - J_3 J_7}$, $C_3 = \frac{J_{13} J_9 - J_{11} J_9}{J_{11} J_{12} - J_{10} J_{13}}$, $\quad C_4 = \frac{J_9 J_{10} - J_9 J_{12}}{J_{11} J_{12} - J_{10} J_{13}}$,

$J_1 = \frac{Gr}{(m_1{}^2 + N^2) Sin(N)}$, $\quad J_2 = \frac{\lambda_0}{m_2{}^2}$, $J_3 = 1 - \gamma m_2$, $\quad J_4 = 1 + \gamma m_2$,

$J_5 = J_2 - \gamma J_1 N$, $\quad J_6 = 1 + \sigma m_2 e^{m_2}$, $J_7 = 1 - \sigma m_2 e^{-m_2}$,

$J_8 = J_1\, N\, \sigma\, Cos\,(N) + J_1\ Sin\,(N) + J_2$, $J_9 = \frac{\lambda_1}{m_3{}^2}$, $J_{10} = 1 - \gamma m_3$,

$J_{11} = 1 - \gamma m_3$, $\quad J_{12} = 1 + \sigma m_3 e^{m_3}$, $J_{13} = 1 - \sigma m_3 e^{-m_3}$,

Appendix 2

$${m_1}^2 = \frac{\beta_1}{1+\beta_1}(H^2 + s^2), \quad {m_3}^2 = \frac{\beta_1}{1+\beta_1}(H^2 + s^2 + Re\, i\omega),$$

$${m_5}^2 = N^2 - \ Pe\, i\omega\ , n_1 = \left(\frac{\beta_1}{1+\beta_1}\right)\frac{\lambda_0}{{m_1}^2},$$

$$n_2 = \left(\frac{\beta_1}{1+\beta_1}\right)\frac{Gr\, N\, \gamma_1}{[(\gamma_1+\gamma_2)\mathrm{NCos(N)}+(1-\gamma_1\gamma_2 N^2)\,\mathrm{Sin}(N)](N^2+{m_1}^2)}\ ,$$

$$n_3 = \left(\frac{\beta_1}{1+\beta_1}\right)\frac{Gr}{[(\gamma_1+\gamma_2)\mathrm{NCos(N)}+(1-\gamma_1\gamma_2 N^2)\,\mathrm{Sin}(N)](N^2+{m_1}^2)}\ ,$$

$$n_4 = \left(\frac{\beta_1}{1+\beta_1}\right)\frac{\lambda_1}{{m_3}^2}\ ,$$

$$C_1 = -\frac{Q_3\, Q_5 - Q_2\, Q_6}{-Q_2\, Q_4 + Q_1\, Q_5}, \qquad C_2 = -\frac{Q_3\, Q_4 - Q_1\, Q_6}{Q_2 Q_4 - Q_1\, Q_5},$$

$$C_3 = -\frac{-Q_{12}\, Q_8 + Q_{11}\, Q_9}{Q_{11}\, Q_7 - Q_{10}\, Q_8}, \qquad C_4 = -\frac{Q_{12}\, Q_7 - Q_{10}\, Q_9}{Q_{11}\, Q_7 - Q_{10}\, Q_8},$$

Q_1=$1 - m_1\gamma\left(\frac{\beta_1+1}{\beta_1}\right)$, $\qquad Q_2$=$1 + m_1\gamma\left(\frac{\beta_1+1}{\beta_1}\right)$,

Q_3=$-n_1 - n_2 + \gamma\left(\frac{\beta_1+1}{\beta_1}\right)n_3 N$, $\qquad Q_4$=$e^{m_1}(1 + \sigma(\frac{\beta_1+1}{\beta_1})m_1)$,

Q_5=$e^{-m_1}(1 - \sigma(\frac{\beta_1+1}{\beta_1})m_1)$,

Q_6=$-n_1 - n_2\mathrm{Cos}[N] - n_3\mathrm{Sin}[N] - \sigma\left(\frac{\beta_1+1}{\beta_1}\right)n_3 N\mathrm{Cos}[N] + n_2 N\mathrm{Sin}[N]\sigma\left(\frac{\beta_1+1}{\beta_1}\right)$,

Q_7=$1 - \gamma(\frac{\beta_1+1}{\beta_1})m_3$,

Q_8=$1 + \gamma\left(\frac{\beta_1+1}{\beta_1}\right)m_{3,}$

Q_9= -n_4,

Q_{10}=$e^{m_3}\left(1 + \sigma\left(\frac{\beta_1+1}{\beta_1}\right)m_3\right)$, $\quad Q_{11}$=$e^{-m_3}\left(1 - \sigma\left(\frac{\beta_1+1}{\beta_1}\right)m_3\right)$,

Q_{12}= -n_4

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