Oscillatory flow of a dusty fluid in a channel with slip condition

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Dedicated to
My respected parents and teachers

whose prayers and support have always been a source of encouragement for me

My caring and supporting Fiance has always given me care and love

Abstract

The objective of this thesis is to analyze the consequences of velocity and thermal slip boundary conditions on the oscillatory flow of dusty nanofluid. The fluid is passing through the vertical permeable channel filled with permeable medium under the impact of magnetic field. By using similarity transformation, the governing partial differential equations are modified in non-dimensional equations, after this the systems of equations are analytically solved by employing separation of variable technique. The physical impact on flow field by different pertinent parameters are offered graphically and discussed. One can observe from the results that concentration of nanoparticles tends to increase the velocity of fluid, velocity of dust particles as well as temperature of the fluid. Velocity slip boundary condition significantly influence the velocity of the fluid rather than velocity of dust particles. Temperature of $EG-Fe_3O_4$ is an increasing function of $Q,$ furthermore the thermal boundary is increased with $Q.$

Contents

Nomenclature

V	Velocity field for fluid phase
$\boldsymbol{N_0}$	Number density of dust particle
$\boldsymbol{V_p}$	Velocity of dust particle in y direction
J	Current density
B	Magnetic field
$\boldsymbol{K_0}$	Stokes constant
R	Darcy resistance
G	Gravitational force
x, y	Cartesian coordinate
C_p	Specific heat at constant pressure
T_0	Left wall temperature
T_w	Right wall temperature
T_f	Fluid initial temperature
A_1	First Rivlin- Erickson tensor
u	Velocity along x-axis
u_p	Particle velocity along x-axis
K	Porous medium permeability coefficient
k	Thermal conductivity
D	Average rate of dust particles
L	Velocity gradient
Q	Radiative heat flux
P	Pressure gradient
I	Identity tensor
T	Nanofluid temperature
Re	Flow Reynolds number
n	Outward normal unit vector
cv	Control volume
cs	Control surface/cross section

e	System energy per unit mass
W_{sur}	Surface work rate
W_{visc}	Viscous work rate
H	Hartmann number
L	Channel length
m	Particle mass parameter
Re	Flow Reynolds number
l	Particle concentration parameter
P	Pressure gradient
B_0	Strength of applied magnetic field
N	Radiation parameter
Gr	Grashof number
s	Permeable medium shape
Greek Letters	
$\boldsymbol{\gamma}$	Navier velocity slip
B	Coefficient of volume expansion due to temperature
μ	Dynamic viscosity
ω	Frequency oscillation
P	Density of the particle
θ	Dimensionless temperature
ϕ	Dimensionless concentration
υ	Kinematic viscosity coefficient
σ	Electrical conductivity
α_{nf}	Thermal diffusivity
ρ_{nf}	Effective density
μ_{nf}	Effective dynamic viscosity
$\left(\rho C_p\right)_{nf}$	Heat capacitance
κ_{nf}	Thermal conductivity

φ	Volume fraction
μ_f	Dynamic viscosity
ρ_f	Density for ferrofluid
ρ_s	Density for base fluid
κ_f	Thermal conductivity of fluid
κ_s	Thermal conductivity of solid particle
α	Thermal slip parameter

LIST OF TABLES

LIST OF FIGURES

Figure No. Title Page No.

Chapter 1

Introduction

In this chapter, we focus on few definitions and also highlight the ruling equations specifying the flow aspects in the channel. Moreover, the basic approach of solution technique like separation of variables is presented in detail.

Fluid mechanics relates to the subject of fluid at rest as well as in flow. Fluid is mainly categorized as ideal and real fluid. The viscous attributes are minimal in ideal fluid whereas real fluid possesses advanced level of viscosity. When viscous stresses of fluid arise from its flow and its local strain rate is linearly proportional to viscous stresses, then this fluid is a Newtonian fluid. But when a fluid does not satisfy the viscosity's law referred as "Newton's law of viscosity" then that fluid is named as a non-Newtonian fluid.

Fluid flow enclosed by channels occurs in engineering, daily life and science. Flow in restrained channels, like region enclosed by parallel plates, is linked with strong boundaries. Mostly all restrained channels in engineering practice have circular or rectangular cross sectional region. Practical applications include nuclear reactors, heat exchangers, solar collectors, micro-electronic equipment's and high intensity electric transmission cables.

Mixture of fluid with dust substance is known as dusty fluid. The existence of dust particles has some positive effects on the flow of the fluid. This analysis is essential in areas including commercial toxins, smoke releases form autos, arrival of effluents from cooperation, soothing impacts of force, air systems and perhaps composition of raindrops.

The medium that consist of interconnected pores (voids), is named as a permeable medium. These pores are usually mixed with a fluid or a gas. A permeable medium is characterized by its consistency. The study of more general behavior of permeable medium involving the information of the solid frame is named as poro-mechanics. Alternative assets of the channel, e.g. porousness, electrical phenomenon, are often obtained from the different assets of its strong, fluid and gas constitution, medium consistency and also porous composition, but this sort of derivation is sometimes complex even if the concept of porosity is easy for a poro-elastic medium. Additionally, there is a concept of closed consistency i.e. the pores basic assessable to flow. Several common substances i.e. rock and soil, crude reservoirs, zero life, biological tissues are some examples and bones, wood, cork, and synthetic materials are often treated as permeable medium. The idea of permeable medium is used in several areas of engineering filtrations, applied sciences, mechanics, civil and rock mechanics, engineering, crude engineering, geo-sciences and petroleum geology. Fluid flow through permeable medium is an interesting subject of research.

Nanofluids (NFs) are the mixtures of nano-sized solid particles with the base fluid (BF). The mixing of tiny strong particles in the energy exchange fluids can enhance their thermal conductivity. NFs have particular chemical and physical properties. Choi et al. [1] demonstrated that fluid thermal conductivity can be doubled with the addition of nanoparticles in it. This property is useful in obtaining more energy. Nanofluids are the highly efficient heat transfer fluids prepared by dispersing nanoparticles less then100 *nm* in diameter in conventional fluids. In many industrial fields like transportation, microelectronics, thermal therapy for cancer treatments etc, improved thermal behavior of nanofluids is of vital importance. There exist some special types of NFs known as ferrofluids (FFs) that are mixture of suspensions of ferroparticles (FPs) and BF. These FPs are cobalt ferrite $\left(CoFe_2O_4\right),$ magnetite $\left(Fe_3O_4\right),$ and many other materials with iron in them. The strength of magnetic resources of this type of fluid can be adjusted by their concentration and size of FPs. Some of the applications of FFs are energy conversion devices, novel pumps and accelerometers.

The flow in which the fluid moves under the impact of magnetic field is called magnetohydrodymics (MHD) flow. The research of MHD flows with buoyancy and viscous dissipation has great significance in industry and geothermal applications. In most of the cases, the fluid flow has more than one phase and is highly affected by the magnetic field. The time-dependent MHD free convection flow governed by the effect of suction/blowing is also a great theme.

Suction/blowing has an important role in the field of space sciences and aerodynamics. For instance, in the model of hydrodynamics thrust bearings and compressor. Suction is also used in chemical process to remove reactants, to transform the raw material into a desired set of products and infusion is used to add reactants. In the light of this motivation, numerous researchers have considered the permeable channel issue with suction and infusion under various physical conditions.

Heat exchange is a transfer of energy that is associated with the temperature variation within a channel or between the walls of channel. The transmission of heat in the fluid flowing within a channel is among the essential aspects of analysis in technological innovations and mechanics. Industrially, the impacts of radiation on heat exchange have become more significant. Radiation effects becomes more effective at high operating temperature. Mostly used heat exchange fluids included water, engine oil and ethylene glycol have low thermal conductivity. Heat transport in unsteady laminar flows has numerous daily life applications, especially flows in a porous channel with permeable walls, which include aerodynamic heating, medical devices, chemical industry, and polymer technology. It possesses substantial amount of thermal engineering utilizations in thermal insulation technology, water mobility in geothermal reservoirs, electronics cooling and heat pipes.

Velocity slip is the distinction among air conveying velocity and particle conveying velocity. The no-slip boundary condition refers to the speed of the fluid layer in direct contact with the boundary is identical to the velocity of this boundary. The slip boundary condition refers to the disconitinuty in the velocity function. There is a slip between the fluid and the boundary.

The behavior of heat generation/absorption in fluids is very significant in sight of many issues like handling chemical reactions. Many researchers have developed their interest in investigating the flow of heat generating/absorbing fluid as a result of this the temperature variations are multiplied, the volumetrically heat generation/absorption term could increase the effect on the heat exchange flow.

1.1 Basic definitions and preliminaries

This section contains standard definitions and basic laws which are helpful in understanding the works in the coming chapters.

1.1.1 Fluid

The substance that deforms continuously under the impact of shearing forces is referred as fluid. The fluids are classified into two categories namely, liquids and the gases. Fluids do not have definite shape. Blood, paint, oil and water are some of examples of fluid.

1.1.2 Fluid mechanics

The main class of mechanics which studies the impacts of fluid statistics or dynamics and illustrate all the fluid properties on boundaries is known as fluid mechanics.

1.1.3 Fluid Statics

The sub-class of fluid mechanics which requires to analyze the conditions when fluid is at rest is called fluid statistics.

1.1.4 Fluid dynamics

The sub-class of fluid mechanics which is used to analyze the conditions when the fluid is in motion is called fluid dynamics.

1.1.5 Flow

Flow is characterized as a material that deforms smoothly and fluently under the effects of various kinds of forces. Flow is further divided into two major subclasses.

1.1.6 Laminar flow

When the fluid flows in regular paths, with no interruption between the layers is known as laminar flow.

1.1.7 Turbulent flow

When the fluid particles have irregular velocity in the flow field, turbulent flow is obtained.

1.1.8 Shear stress

The external force exerted on a material parallel to the surface of unit area is categorized as shear stress. Mathematically

$$\tau = \frac{F}{A},$$

where τ denotes the shear stress, F stands for applied force and A is area.

1.1.9 Density

The density is mass of material per unit volume. This is expressed mathematically as

$$\rho = \frac{m}{V}.$$

The unit of ρ is $\frac{kg}{m^3}$.

1.1.10 Viscosity

The primary characteristic of the fluid that measures the fluid's resistance to flow when numerous forces are acting on it. Mathematically we can write is as follows

$$\text{viscosity}(\mu) = \frac{\text{shear stress}}{\text{gradient of velocity}}.$$

1.1.11 Dynamic viscosity

Dynamic viscosity is defined as the measure of fluid resistivity to flow. Its unit is kg/ms

1.1.12 Kinematic viscosity

A parameter often appear in equation of motion that is obtained by dividing absolute viscosity and fluid mass density. Mathematically we write

$$\upsilon = \frac{\mu}{\rho}.$$

1.1.13 Magnetohydrodynamics

It describes the magnetic effect of electrically conducting fluids. The word magnetohydrodynamics is the combination of words magneto mean magnetic, hydro means water or liquid and dynamics refer to the motion of an object by forces.

1.1.14 Skin friction coefficient

Skin friction $\left(C_f\right)$ is a resistance at a surface that occurs when an object moves in a fluid. The C_f is expressed as

$$C_f = \frac{\tau_w}{q}.$$

Here surface shear stress is τ_w and free stream dynamic pressure is q .

1.1.15 Newton's law of viscosity

The fluid that opposes the relative motion between the two layers of the fluid between shear stress and velocity gradient. The fluid is temperature dependent: In liquids it decreases with an increase in temperature while for gases it shows opposite behavior. The relation between shear stress and velocity is defined by Newton's law. Mathematically we write

$$\tau_{yx} = \mu\left(u_y\right).$$

In which τ_{yx} indicates the shear force applied on the fluid's element and μ indicates the proportionality constant.

1.1.16 Porous surfaces

It is a material which made out of pores, over which fluid or gas can travel through. Few examples are biological tissues, cork and rocks, sponges, fabrics ceramics and foams are also gathered for the purpose of porous media.

1.1.17 Porosity

The measure of spongy space in a porous substance is known as porosity.

1.1.18 Permeability

It is the strength of a porous substance to allow fluid to travel through it. Those materials which have low porosity are minor permeable while materials having large pored are easily permeated and have high porosity.

1.2 Dimensionless parameters

1.2.1 Prandtl number (Pr)

It represents the ratio between momentum to the thermal diffusivities. Mathematically

$$\Pr = \frac{\upsilon}{\alpha},$$

In which υ for the momentum diffusivity or kinematic viscosity and α for thermal diffusivity. Thermal diffusivity dominates for small Pr values whereas for large Pr values viscous diffusivity dominates.

1.2.2 Reynolds number (Re)

It describes inertial to viscous forces. Mathematically, this number is expressed as

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}}.$$

Reynolds number are utilized to describe distinct flow behaviors within a similar fluid. laminar flow arises at small Reynolds number, in which we can note that viscous effects are eminent.

1.2.3 Local Nusselt Number

The proportion of convective to conductive heat exchange between solid boundary and moving fluid is said to be local nusselt number. It is written as

$$Nu = \frac{h.l}{k}.$$

The effect of conduction and convection is same when local Nusselt number is considered to be one. Here l is characteristic length, the thermal conductivity is k, and convective heat exchange coefficient is h.

1.2.4 Hartmann number

It shows the connection between viscosity and frictional forces induced by magnetism. It plays vital role in MHD. Hartmann number is the ratio of electromagnetic to the viscous forces i.e.

$$Ha = B_o L = \sqrt{\frac{\sigma}{\mu}}$$

1.3 Basic laws

The fundamental laws that are used for the flow applications in the sub sequential analysis are given below. The continuity equation, the conservation laws of momentum and energy are the fundamental laws in fluid mechanics.

1.3.1 Equation of continuity

The mass conservation principle deals with the continuity equation

$$\rho_t + \nabla.\left(\rho \mathbf{V}\right) = 0, \tag{1.1}$$

which is the continuity equation for a compressible fluid. For steady flow

$$\rho_t = 0. \tag{1.2}$$

Then the above Eq. (1.1) becomes

$$\nabla.\left(\rho \mathbf{V}\right) = 0. \tag{1.3}$$

This is called equation of mass conservation. If ρ is also constant, then we get

$$\nabla.\mathbf{V} = 0.$$

1.3.2 Conservation of momentum equation

The conservation of momentum is based on law of conservation of linear momentum.

$$\rho\left(\frac{d\,\mathbf{V}}{dt}\right) = -\text{div}.\pi + \rho\mathbf{g}. \qquad (1.4)$$

The surface force is due to the stresses which are summation of the viscous stresses τ_{ij} plus the hydrostatic pressure on the sides of control surface that comes from the motion of the velocity gradient i.e.

$$\pi_{ij} = -\rho\delta_{ij} + \tau_{ij},$$

So,

$$\nabla.\pi = -\nabla p + \nabla.\tau.$$

After substituting the above relation in Eq. (1.4), we get

$$\rho\left(V\right)_t = \rho g - \nabla p + \nabla.\tau \qquad (1.5)$$

1.3.3 Conservation of energy equation

According to this law "energy can neither be created nor destroyed".
Energy equation is given as

$$Q - W_{sur} - W_{visc} = \frac{\partial}{\partial t}\left(\int_{cv} e\rho dV\right) + \int_{cs}\left(e + \frac{p}{\rho}\right)\rho\left(V.n\right)dA \qquad (1.6)$$

where $W_{sur} = 0$ because there can be no infinitesimal shaft protruding into the control volume. The right hand side for tiny element becomes,

$$Q - W_{visc} = \left(\frac{\partial}{\partial t}\left(\rho e\right) + \frac{\partial}{\partial x}\left(\rho u\xi\right) + \frac{\partial}{\partial y}\left(\rho v\xi\right) + \frac{\partial}{\partial z}\left(\rho w\xi\right)\right)dxdydz, \quad (1.7)$$

where $\xi = e + \dfrac{\rho}{p}$. Using continuity equation the above equation becomes

$$Q - W_{visc} = \left(\rho \frac{de}{dt} + V.\nabla p + \rho \nabla .V \right) dxdydz. \tag{1.8}$$

To evaluate Q, we use Fourier law of conduction. Adding the inlet terms and subtracting outlet terms, we get

$$Q - W_{visc} = \left(\frac{\partial}{\partial x}(q_x) + \frac{\partial}{\partial y}(q_y) + \frac{\partial}{\partial z}(q_z) \right) dxdydz = -\nabla .q dxdydz. \tag{1.9}$$

Using Fourier's law we have

$$Q = \nabla .(k \nabla T) dxdydz \tag{1.10}$$

The net viscous work rate after outlet terms are subtracted from the inlet terms, which becomes

$$W_{visc} = -\nabla .(V.\tau_{ij}) dxdydz.` \tag{1.11}$$

By substituting (1.10) and (1.11) into Eq. (1.9), eliminating $\nabla .\tau_{ij}$ and by using linear momentum equation we get the final differential form of energy equation as

$$\rho \frac{du}{dt} + \rho (\nabla .v) = \nabla .(k \nabla T) + \varphi, \tag{1.12}$$

where

$$\varphi = \tau_{ij} \frac{\partial u_i}{\partial x_j}.$$

1.4 Solution method

Different circumstances in technological and engineering world are mainly nonlinear and successfully demonstrated with the help of ordinary differential equations (ODEs) or partial differential equations (PDEs). Many techniques are used to find the solutions of ODEs and PDEs. The nonlinear PDEs are first converted to nonlinear ODEs by using similarity transformations.

1.4.1 Separation of variables

Separation of variables (SOV) is used for solving ODEs and PDEs. SOV technique is very useful method applicable in nonlinear dynamical models. The solution is represented in not more than two terms.

Example

Suppose a linear ODE

$$\frac{\partial f(u,v)}{\partial u} + \frac{\partial f(u,v)}{\partial v} = 0 \tag{1.13}$$

with initial condition (IC)

$$f(0) = 1.$$

The first step is to suppose that the solution of the differential equation $f(u,v)$ can be written as the product of functions u and v

$$f(u,v) = h(u)\,g(v). \tag{1.14}$$

In second step, we substitute $f(u,v)$ in Equation (1.13)

$$\frac{\partial h(u)\,g(v)}{\partial u} + \frac{\partial h(u)\,g(v)}{\partial v} = 0. \tag{1.15}$$

$$g(v)\frac{\partial h(u)}{\partial u} + h(u)\frac{\partial g(v)}{\partial v} = 0. \tag{1.16}$$

The third step involves reorganizing the terms of Eq. (1.15) so all terms in x and y are grouped together. There is no universal method for this step. In this example, we separate variables by dividing all terms by $g(u)h(v)$, but in general we need to Figureure out how to separate variables for the particular equation you are solving:

$$\frac{1}{g(u)}\frac{\partial g(u)}{\partial u} + \frac{1}{h(v)}\frac{\partial h(v)}{\partial v} = 0 \tag{1.17}$$

In the fourth step, we recognize that Eq. 1.16 is the sum of two terms (it would be three if we were solving a problem in 3 dimensions), and each term depends on one variable only. In this case, the first term is a function of x only, and the second term is a function of y

only. The term $\frac{1}{g(u)}\frac{\partial g(u)}{\partial u}$ cannot be a function of u, and the term $\frac{1}{h(v)}\frac{\partial h(v)}{\partial v}$ cannot be a function of v

$$\frac{1}{g(u)}\frac{\partial g(u)}{\partial u}=c_1, \tag{1.18}$$

$$\frac{1}{h(v)}\frac{\partial h(v)}{\partial v}=c_2, \tag{1.19}$$

This step transforms a PDE into two ODEs. In general we will have one ODE for each independent variable. In this particular case because the two terms need to add up to zero we have

$$c_1=-c_2.$$

In the fifth step, we will solve the two ODEs. We will get $g(u)$ from Equation 1.18 and $h(v)$ from Equation 1.19. Both solutions will contain arbitrary constants that we will evaluated using initial or boundary conditions if given. In this case, the two equations are mathematically identical, and are separable 1st order ordinary differential equations. The solutions are

$$f(u,v)=g(u)h(v)=Ae^{c_1 u}Be^{-c_1 v}=De^{c_1(u-v)},$$

$$g(u)=Ae^{c_1 u}, \tag{1.20}$$

$$h(v)=Be^{-c_1 v}. \tag{1.21}$$

In step six we combine the one variable solutions to obtain the many variables solution we are looking for

where D is a constant.

Chapter 2

Literature Review

Dusty fluid flow is a subject of great enthusiasm in recent decade. Several researchers worked on heat exchange and flow of dusty fluid with different geometries. Saffman [2] did an analysis on the flow of uniformly spread dust particles in a gas. Vajravelu and Nayfeh [3] examined the results of suction on the progression of electrically conducting liquid containing even dissemination of dust particles. Turkyilmazoglu [4] gave the exact solution for the movement of a viscous dusty liquid over extending or contracting sheets. Makinde and Chinyoka [5] talked about the heat exchange and MHD flow of dusty fluid in a channel along with navier slip condition and variable physical properties. An investigation on the fluid flow carrying dust particles enclosed by two equal plates was investigated by Parakash et al. [6]. In an article, Jalil et al. [7] focused on the flow of dusty liquid over an elastic surface. Gireesha et al. [8] explored the unsteady dusty fluid flow with the impact of pulsatile pressure gradient and uniform field. Kulandaival [9] explored the flow of a dusty gas through a vertical medium. Debnath and Ghosh [10] studied on unstable MHD flow between two oscillating plates of a dusty fluid. Saidu et al. [11] investigated dusty fluid convective flow and thermal exchange by considering dust particle volume fraction.

The research in mixed convection heat exchange in vertical permeable medium has gained the interests of many researchers. The applications of mixed convection flow is used in science as in astrophysics and geophysics. Mixed convection is the flow under the result of pressure gradient and thermal buoyancy. Acvi and Ayndin [12] worked on combined convective flow in a vertical medium. Aydin and Kaya [13] discussed the role of suction/infusion and attractive field on the flow about a vertical medium. Hayat et. al. [14] demonstrated the impacts of thermal radiation in a Jeffrey fluid. The problem of MHD mixed convection flow and mass exchange in a vertical permeable medium is demonstrated by Raveendra Nath [15]. O.D.Makinde [16] discussed the MHD mixed convection stagnation point flow in a vertical permeable medium.

For the improvement of heat exchange in ordinary cooling liquids i.e. oil, water, ethylene glycol, Choi [1] presented NFs in which NPs are added in a base fluid. several researchers studied the flow of NF and FF. Sheikholislami et al. [16] demonstrated the impact of attractive field in the NF flow in a permeable medium. Hang and Pop [17] evaluated the fully established blended convection flow nanofluid in a vertical medium. Domairry et al. [18] explored the natural convection between two infinite parallel vertical plates of a non-Newtonian $Cu-H_2O$ nanofluid. Kashif et al. [19] regarded the mass exchange flow with the impact of Re in the region of metallic oxide NPs among systematically moving porous plates. Khan et al. [20] discussed the problem of FFs along a flat plate. They considered FPs of different types e.g. $Fe_3 0_4$, $CoFe_3O_4$ and $Mn-ZnFe_2O_4$. They considered two kinds of liquids as BF i.e. kerosene oil and water. Mohan Krishna et al. [21] evaluated the impact of radiation on an unstable natural convective flow over a vertical plate of an MHD nano fluid by considering the impact of heat source.

Many researchers have considered the permeable channel problem with suction/ infusion under different physical conditions. Sadri and Baba elahi [22] conducted an evaluation on laminar layer flowing over a permeable flat plate with suction/infusion ate the wall. Ali [23] discussed the influence of suction/infusion on free convection boundary layer. Torda [24] analyzed the boundary layer flow with the suction/infusion effect. Labropulu et al. [25] depicts that to get rid of reactants suction is applied and for cooling the surfaces infusion is used. Attia [26] described the influence of suction/infusion on Couette flow. Ahmed and Khatun [27] explains a theoretical assessment on MHD oscillatory flow with suction/infusion. Magyari and Chamkha [28] discussed the analysis of the heat generation or absorption impact. Mutuku-Njane and Makinde [29] presented the impact of MHD on physical phenomenon flow with slip boundary condition along with suction. Rundora and Makinde [30] demonstrated the impacts of suction/infusion during a permeable medium on unsteady reactive thermal consistency. Makinde and Chinyoka [31] examined the results of suction/infusion through a channel on non-Newtonian fluid flow

Deswita et al. [32] illustrated the impact of suction/infusion on constant of boundary-layer flow. Hamid et al. [33] evaluated the results of heating, viscous dissipation, suction/infusion radiation over a flat surface. Makinde and Chinyoka [34] investigated the numerical answer of transient pressure flow with suction/infusion channel. Berman [35] depicted an exact solution for the channel flow taking into consideration the uniform suction/infusion at the boundary wall of the channel. Shojaefard et al. [36] demonstrated fluid flow control on the surface of subsonic airfoil by suction/infusion. Ishak et al. [37] demonstrated that in mass exchange cooling, suction/infusion of a fluid will change the flow field and so effect the heat exchange rate form the plate. Griffith and Meredith [38] examined the flow through a permeable plate of a viscous fluid with uniform suction. Jena and Mathur [39] investigated free convection within the fluid flow subject to uniform suction/infusion. Al-Sanea [40] reported combined convection heat exchange through a vertical plate with suction/infusion. Mass exchange and free convection flow over a vertical medium with suction/infusion are investigated by Takhar et al. [41]. Cortell [42] demonstrated the impacts of suction/infusion, on flow and heat exchange over an infinite permeable plate. Impact of suction/infusion on unsteady free convection Couette flow and heat exchange of viscous fluid in vertical permeable plate is discussed in investigation of Jha et al. [43]. Generally, suction tends to boost the heat exchange co-efficient and skin-friction whereas infusion acts within the opposite manner. Shojaefard et al. [44] studied the suction/infusion to manage fluid flow on the surface of craft.

Research into heat generating/absorbing fluid flow has been of excellent concern. Many researchers considered heat absorption/generation in their work. In perspective of many physical issues, those involving chemical reactions and related to dissociating liquids, the research of heat generation / absorption impacts in moving fluids is essential. Heat absorption/generation performs a vital act in several applications i.e. applications in nuclear energy etc. Chamkha [45] demonstrated the results of heat absorption/generation in MHD flow of vertical permeable channel with chemical reaction. Ogulu [46] has acquired an analytical solution for radiation and heat absorption and mass exchange on polar fluid flow. The combined convection flow placed in a permeable channel was discussed by Chamkha [47].

The slip-flow regime phenomenon has drawn the attention of several researchers because of its broad implementation. In this modern era of science technology and vast industrialization the slip flow regime is very essential. The particle adjacent to a solid surface no longer requires the surface velocity in practical applications. The surface particle slides along the surface with a finite tangential velocity. Eegunjobi and Makinde[48] described the literature and implementation in different geometries of the slip stream. Rundora and Makinde[49] have recently evaluated Navier's impact on slip and variable viscosity through a permeable channel with asymmetric convective boundary conditions. Ghosh et al. [50] discussed about the convective and absolute instabilities in a vertical channel.

Chapter 3

Fluid particle interaction in oscillatory flow through vertical walls

In this chapter we investigated the time dependent flow of an incompressible dusty fluid in a permeable medium surrounded by two vertical plates. The governing equations are non-dimensionalised and solved analytically by separation of variable method. The solution's focuses on characterizing the fluid by noticing the influence of *Re*, *N*, *H* and *Gr* on flow fields. The obtained results are presented through graphs and discussed in detail. This chapter is also the review of the work done by Prakash et al. [6].

3.1 Mathematical analysis

The laminar flow is considered under the impact of radiative heat flux (RHF) in a permeable medium, filled with incompressible dusty fluid. The magnetic field strength is perpendicular to the plate. The medium is drawn along the x-axis and the walls are perpendicular to y-axis. The fluid temperature on both walls is different. T_f is the fluid initial temperature. T_0 is the left wall temperature whereas T_w is the right wall temperature. Re is very small and hence the induced magnetic field becomes insignificant.

We consider a Boussinesq model for incompressible fluid and the constitutive equations using above assumptions are given as:

$$(u)_t = -\frac{1}{\rho}(P)_x + \upsilon (u)_{yy} - \frac{\upsilon}{K}u + \frac{N_0 k_0}{\rho}(u_p - u) - \frac{\sigma_e}{\rho}B_0^2 u + g\beta (T - T_0), \quad (3.1)$$

$$(u_p)_t = k_0 (u - u_p), \quad (3.2)$$

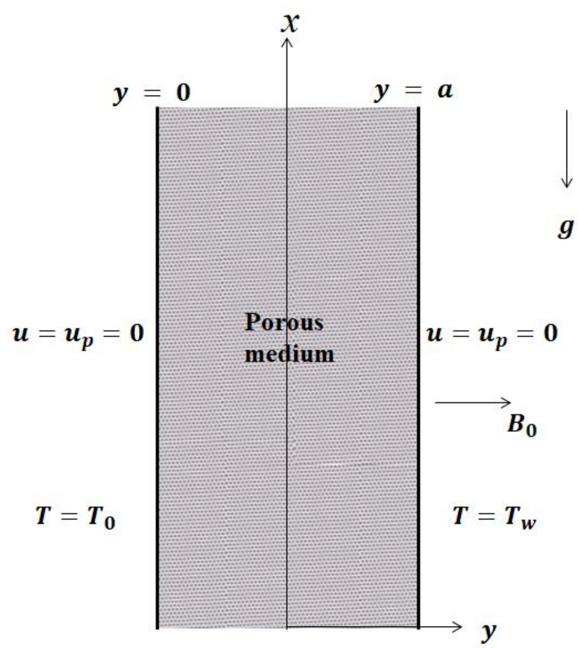

Figure. 3.1: Geometry of the channel

$$(T)_t = \frac{k}{\rho c_p}(T)_{yy} - \frac{1}{\rho c_p}(q)_y. \tag{3.3}$$

The expression for RHF with the assumption of optically thin fluid having small density is given as

$$(q)_y = 4\alpha^2 (T_0 - T), \tag{3.4}$$

where α is the mean radiation absorption coefficient.

The initial and boundary conditions (IBCs) are

$$u(y,0) = 0, \qquad u_p(y,0) = 0, \qquad T(y,0) = T_f, \tag{3.5}$$

$$u(a,t) = 0, \qquad u_p(a,t) = 0, \qquad T(a,t) = T_w = T_0 + (T_f - T_0)e^{iwt}, \tag{3.6}$$

$$u(0,t) = u_p(0,t) = 0, \qquad T(0,t) = T_0. \tag{3.7}$$

Introducing the following dimensional variables to simplify the flow equations.

$$\bar{x}=\frac{x}{a},\quad \bar{y}=\frac{y}{a},\quad \bar{u}=\frac{u}{U},\quad \theta=\frac{T-T_0}{T_f-T_0},\quad \bar{t}=\frac{tU}{a},\quad Da=\frac{K}{a^2},$$

$$M=\frac{\upsilon}{K_0a^2},\quad l=\frac{N_0K_0a^2}{\rho\upsilon},\quad \mathrm{Re}=\frac{Ua}{\upsilon},\quad \mathrm{Pr}=\frac{\upsilon\rho c_p}{k},\quad N^2=\frac{4\alpha^2a^2}{k}, \tag{3.8}$$

$$\bar{u}_p=\frac{u_p}{U},\quad \mathrm{Gr}=\frac{g\beta(\mathrm{T}_f-\mathrm{T}_0)a^2}{\upsilon U},\quad \bar{\mathrm{P}}=\frac{aP}{\upsilon\rho U},\quad H^2=\frac{a^2\sigma_e B_0^2}{\upsilon\rho},\quad s^2=\frac{1}{Da}.$$

The dimensionless equations with boundary conditions (BCs) are

$$\mathrm{Re}(u)_t=-(P)_x+(u)_{yy}-\left(H^2+s^2+l\right)u+l\,u_p+Gr\theta, \tag{3.9}$$

$$\mathrm{Re}\,M\,(u_p)_t=u-u_p, \tag{3.10}$$

$$\mathrm{Re}\,\mathrm{Pr}(\theta_t)=(\theta)_{yy}+N^2\theta, \tag{3.11}$$

subject to conditions

$$u(y,0)=u_p(y,0)=0,\quad \theta(y,0)=1, \tag{3.12}$$

$$u(1,t)=u_p(1,t)=0,\quad \theta(1,t)=e^{i\omega t}, \tag{3.13}$$

$$u(0,\mathrm{t})=u_p(0,t)=0,\quad \theta(0,t)=0. \tag{3.14}$$

3.2 Analytical Procedure

Here we solve the Eqs. (3.9), (3.10) and (3.11) for pure oscillatory flow by letting

$$\begin{aligned} -P_x&=\lambda e^{i\omega t},\quad u(y,t)=u_0(y)e^{i\omega t},\\ u_p(y,t)&=u_{p0}(y)e^{i\omega t},\quad \theta(y,t)=\theta_0(y)e^{i\omega t}. \end{aligned} \tag{3.15}$$

Putting the values from Eq. (3.15) into Eq. (3.9) – (3.14), we obtain

$$(u_0)_{yy}-m_2^2u_0=-\lambda-Gr\theta_0, \tag{3.16}$$

$$u_{p0}=\frac{u_0}{(1+i\omega\,\mathrm{Re}\,M)}, \tag{3.17}$$

$$(\theta_0)_{yy}+m_1^2\theta_0=0. \tag{3.18}$$

with

$$u_0=(u_p)_0=0, \quad \theta_0=0 \quad \text{on} \quad y=1, \tag{3.19}$$

$$u_0=(u_p)_0=0, \quad \theta_0=0 \quad \text{on} \quad y=0. \tag{3.20}$$

Here

$$m_1=\sqrt{N^2-i\,\omega\,\mathrm{Re}\,Pr} \text{ and } \quad m_2^2=s^2+H^2+i\,\omega\,\mathrm{Re}+\frac{1}{i\,\omega\,\mathrm{Re}\,M}.$$

On solving Eq. (3.18) with BCs (3.19) and (3.20), we find the temperature as

$$\theta(y,t)=\frac{\sin(m_1 y)}{\sin(m_1)}e^{i\omega t}. \tag{3.21}$$

Using Eq. (3.16) along with (3.19) and (3.20), the dusty fluid velocity is obtained as

$$u(y,t)=\begin{pmatrix} \dfrac{Gr}{m_1^2+m_2^2}\left(\dfrac{\sin(m_1 y)}{\sin(m_1)}-\dfrac{\sinh(m_2 y)}{\sinh(m_2)}\right)+ \\ \dfrac{\lambda\sinh(m_2 y)}{m_2^2\sinh(m_2)}\left(\cosh\mathrm{m}_2-1\right)+\dfrac{\lambda}{m_2^2}\left(1-\cosh\mathrm{m}_2\, y\right) \end{pmatrix} e^{i\omega t}. \tag{3.22}$$

From Eq. (3.17), dust particles velocity is expressed as

$$u_p(y,t)=\frac{e^{i\omega t}}{1+\imath\,\omega\,\mathrm{Re}\,M}\begin{pmatrix} \dfrac{Gr}{m_1^2+m_2^2}\left(\dfrac{\sin(m_1 y)}{\sin(m_1)}-\dfrac{\sinh(m_2 y)}{\sinh(m_2)}\right)+ \\ \dfrac{\lambda\sinh(m_2 y)}{m_2^2\sinh(m_2)}\left(\cosh\mathrm{m}_2-1\right)+\dfrac{\lambda}{m_2^2}\left(1-\cosh\mathrm{m}_2\, y\right) \end{pmatrix}. \tag{3.23}$$

The expression for C_f and Nu are given as

$$C_f=\frac{a\,\tau_f}{\rho\,\upsilon\,U}=\left(u\right)_y\big|_{y=1}=\begin{pmatrix} \dfrac{Gr}{m_1^2+m_2^2}\left(m_1\cot(m_1)-m_2\coth(m_2)\right)+ \\ \dfrac{\lambda\coth(\mathrm{m}_2)}{m_2}\left(\cosh(m_2-1)\right)-\dfrac{\lambda}{m_2}\left(1-\cosh(m_2-1)\right) \end{pmatrix} e^{i\omega t}, \tag{3.24}$$

where $\tau_f=\rho\upsilon(u)_y$ at $y=a$. For dusty particles the skin friction is given as

$$C_f = \frac{a\tau_f}{\rho \upsilon U} = \left(u_p\right)_y\Big|_{y=1} = \frac{e^{i\omega t}}{1+i\omega \operatorname{Re} M}\left(\begin{array}{l} \frac{Gr}{m_1^2+m_2^2}\left(\begin{array}{l} m_1 \cot\left(m_1\right) \\ -m_2 \coth\left(m_2\right)\end{array}\right)+ \\ \frac{\lambda \coth\left(\mathrm{m}_2\right)}{m_2}\left(\cosh\left(m_2-1\right)\right) \\ -\frac{\lambda}{m_2}\left(1-\cosh\left(m_2-1\right)\right)\end{array}\right), \tag{3.25}$$

where $\tau_f = \rho\upsilon\left(u_p\right)_y$ at $y=a$. Here Nu is given as

$$Nu = \frac{a\,q_w}{k\left(T_f - T_0\right)} = -\left(\theta\right)_y\Big|_{y=1} = -m_1 \cot\left(m_1\right)e^{i\omega t}, \tag{3.26}$$

here $q_w = -k\left(T\right)_y$ at $y=a$ is heat flux at the right wall.

3.3 Results and Discussion

The non-linear ODEs from (3.16) to (3.18) with the BCs (3.19) and (3.20) are solved analytically using SOV technique. Figures. 3.2–3.16 show the numerical values for both dusty fluid and dust particles. We have used the following parametric values for our numerical computation i.e. H = 0, 1, 2, 3; λ = 0.5; N = 0, 0.5, 1, 1.5, 2, 3; M = 0.5; Re = 1, 3, 4, 5; Gr = 0,1, 2, 3; s = 0, 1, 2, 3; ω = 1 and $\Pr$ = 0.71.

Figure. 3.2 displays the temperature profile effect of N. We noticed the temperature of the dusty fluid increases with an increase in *N*. From Figure. 3.3 it can be seen that an increment in *N* results in the decrement in *Nu* because heat gradient decreases at the walls.

Figure. 3.4 - 3.11 depicts the velocity profiles through the medium. It can be noticed that the nature of velocity profiles are parabolic having zero values at the plates. From Figure. 3.4, we see that the effect of *N* on velocity profile creates an increase in the dusty fluid speed. The increment in dusty fluid velocity is seen in Figure. 3.5 with increasing value of *Gr* because of buoyancy force. Figure. 3.6 describes the decrement in the velocity profile with an increment in *H* because of rise in magnetic field. Figure. 3.7 describes a decrement in $u_p\left(y,t\right)$ with an increment in porosity parameter.

Figure. 3.8 and Figure. 3.9 depict the enhancement in dusty particles velocity with enhancing values of *N* and *Gr*. In Figure. 3.10 and 3.11 the dust fluid velocity profiles are reduced with the enhancement in *H* and *s*. It can be founded that the dust particles are moved inside the fluid and an increment in the liquid velocity will make an increment in the particles velocity whereas a reduction in the fluid velocity results in decrement of the particle velocity

Figure. 3.12 indicates the C_f at the wall enhances with the enhancement in the *N*. In Figure. 3.13, we see that the C_f for particle decreases with the rise in *Re* and *H*. Same pattern for reduction in C_f is appeared in Figure. 3.14 with an increment in *s* because of decrement in k . Figures. 3.15– 3.16 shows the consequences for parameter increment on C_{fp} . An increment in C_{fp} results in an increment in *N* and *Gr* however C_{fp} decreases with an increment in *Re*, *H*, and s .

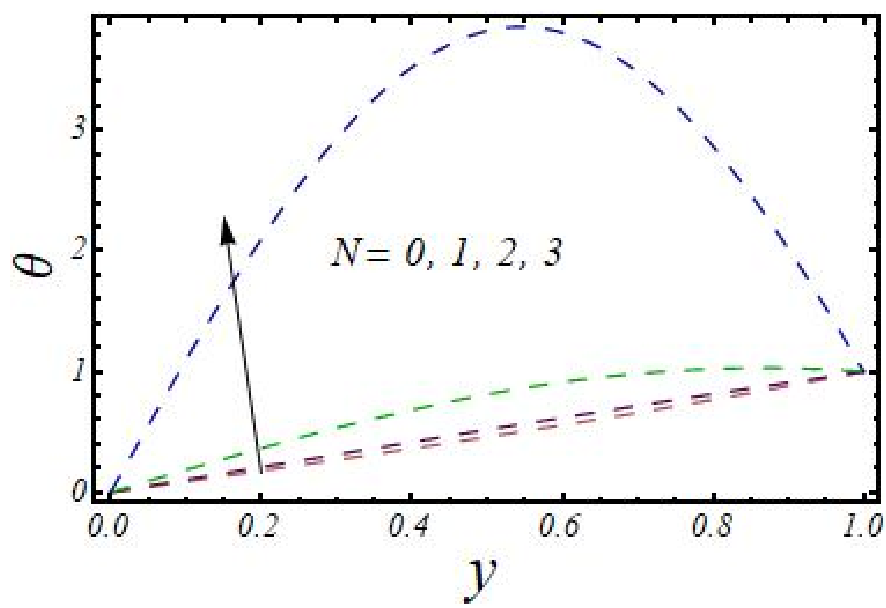

Figure 3.2: θ with *N*.

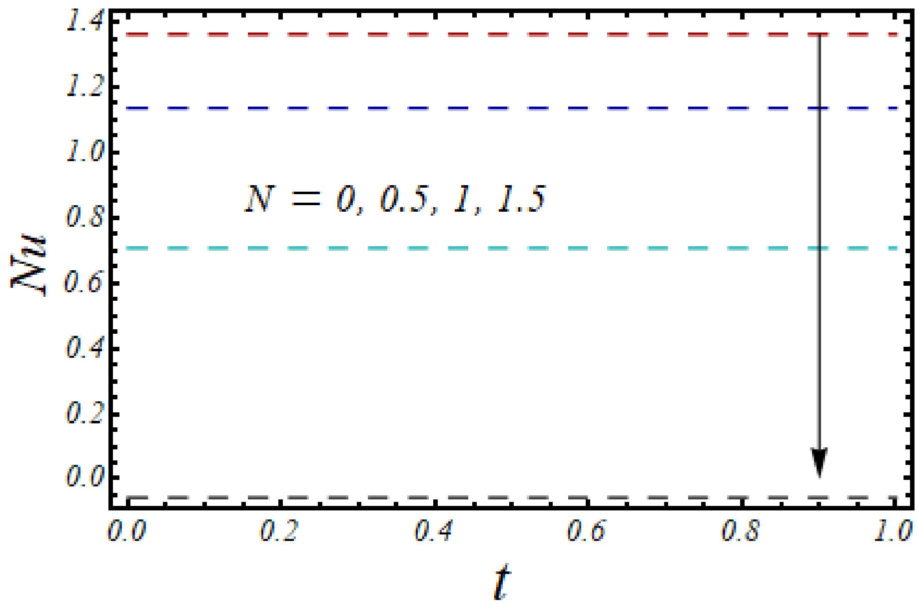

Figure 3.3: Nu versus t with N.

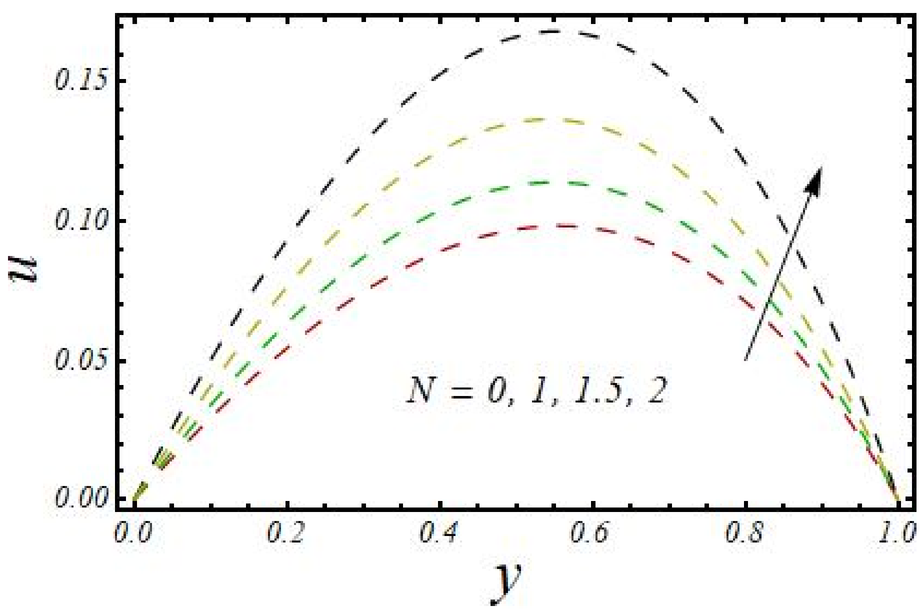

Figure 3.4: u with N.

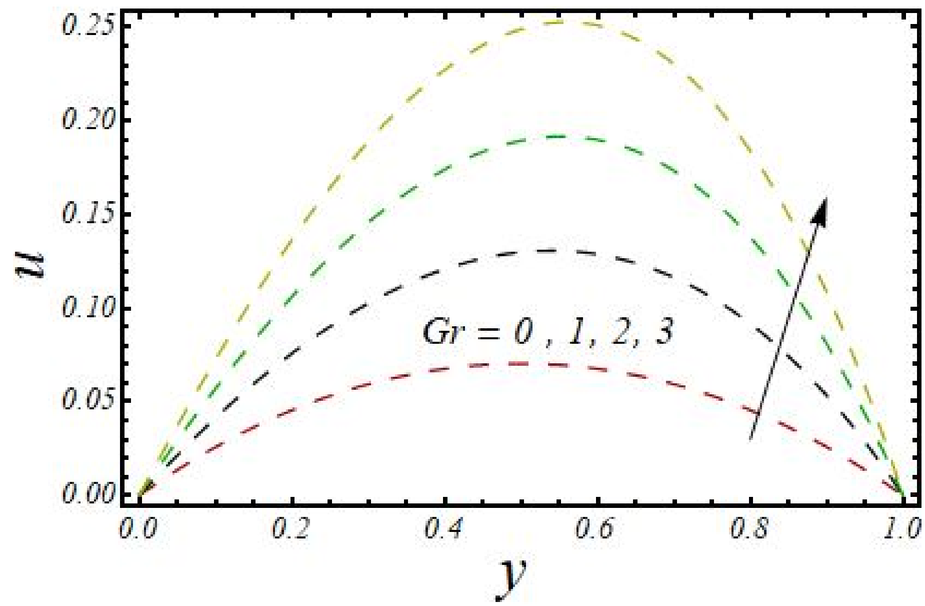

Figure 3.5: u with Gr.

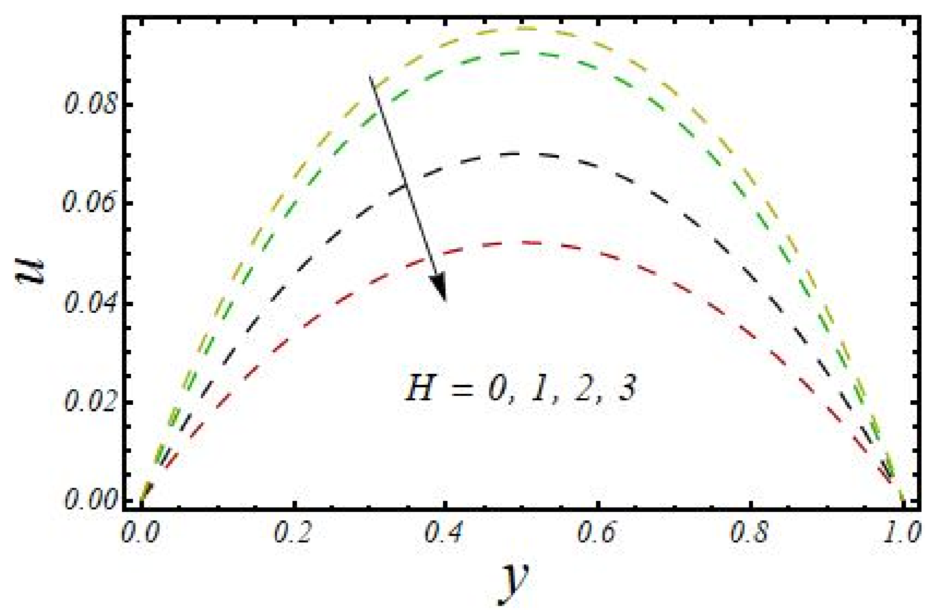

Figure 3.6: u with H.

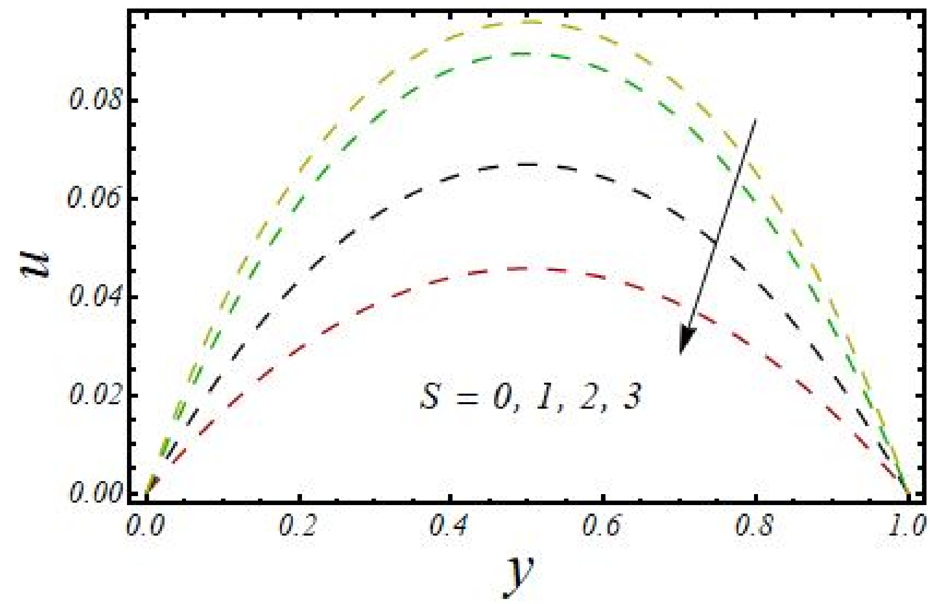

Figure 3.7: u with S.

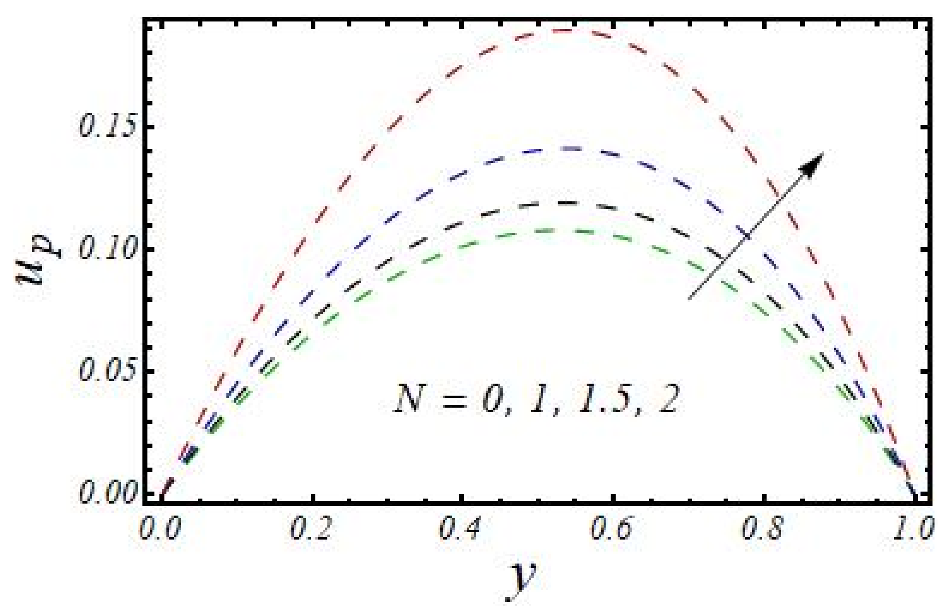

Figure 3.8: u_p with N.

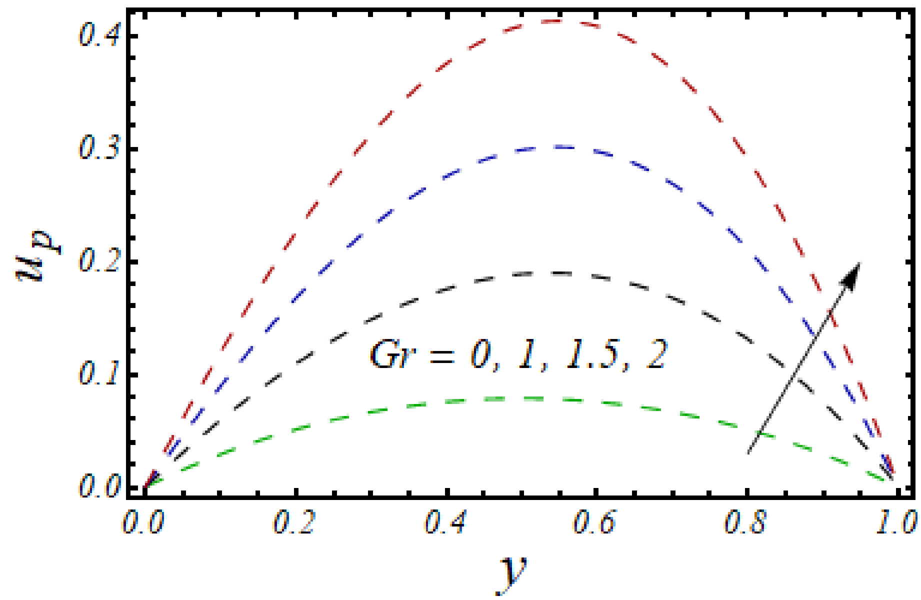

Figure 3.9: u_p with Gr.

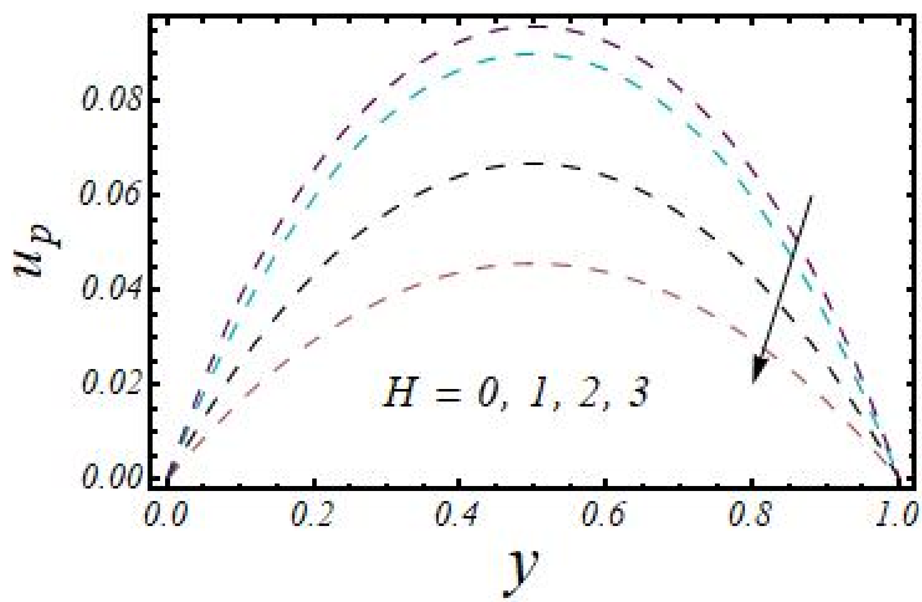

Figure 3.10: u_p with H.

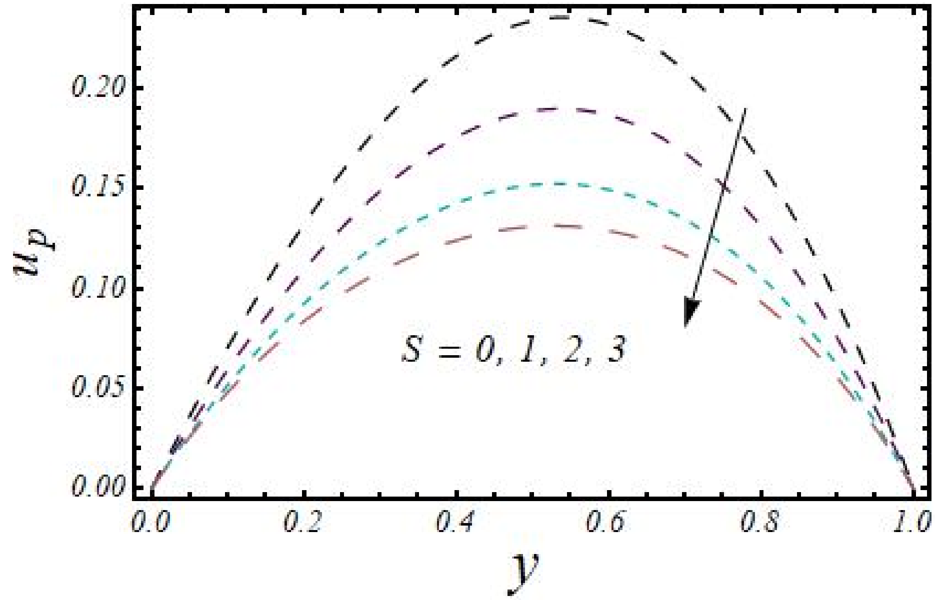

Figure 3.11: u_p with S

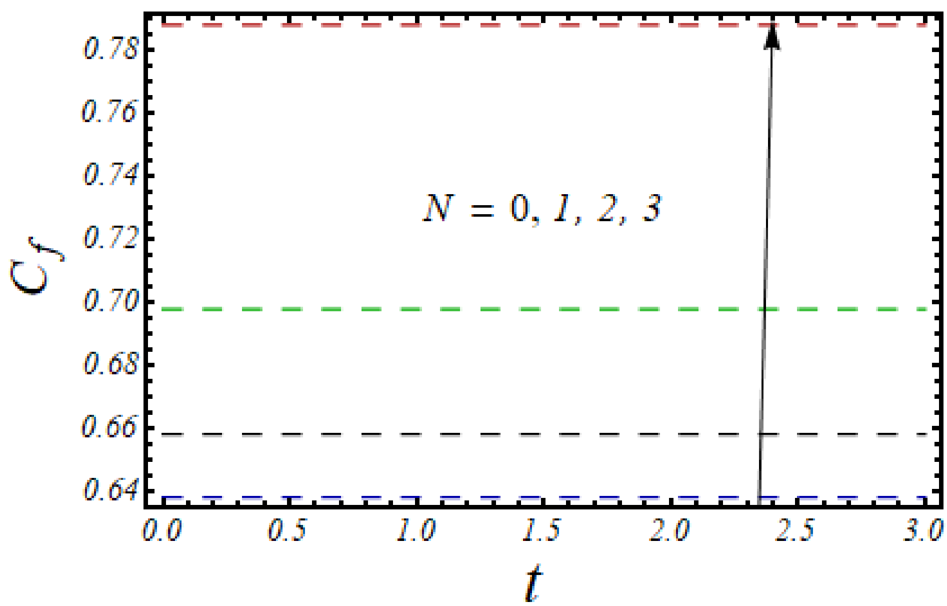

Figure 3.12: C_f versus t with N.

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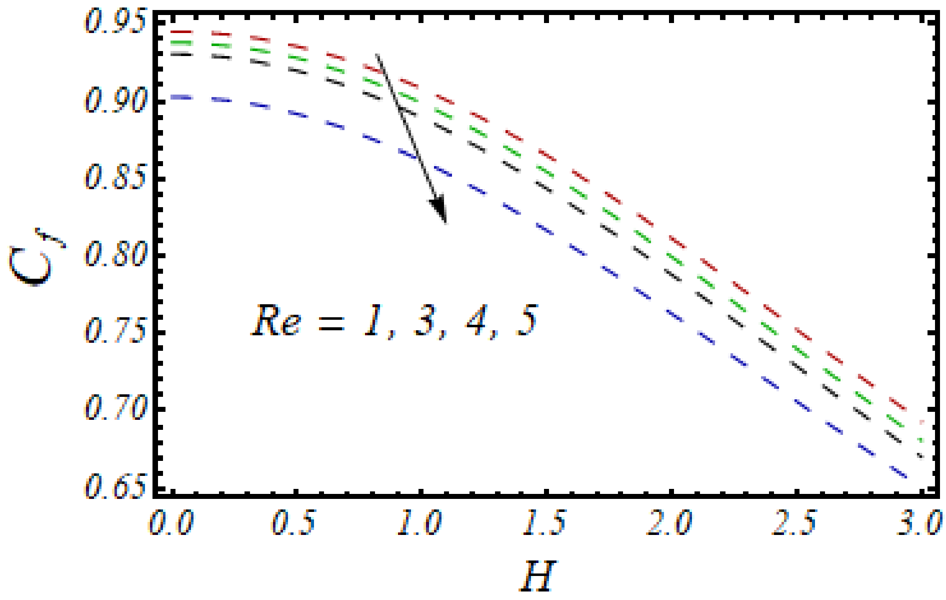

Figure 3.13: C_f versus H with *Re*.

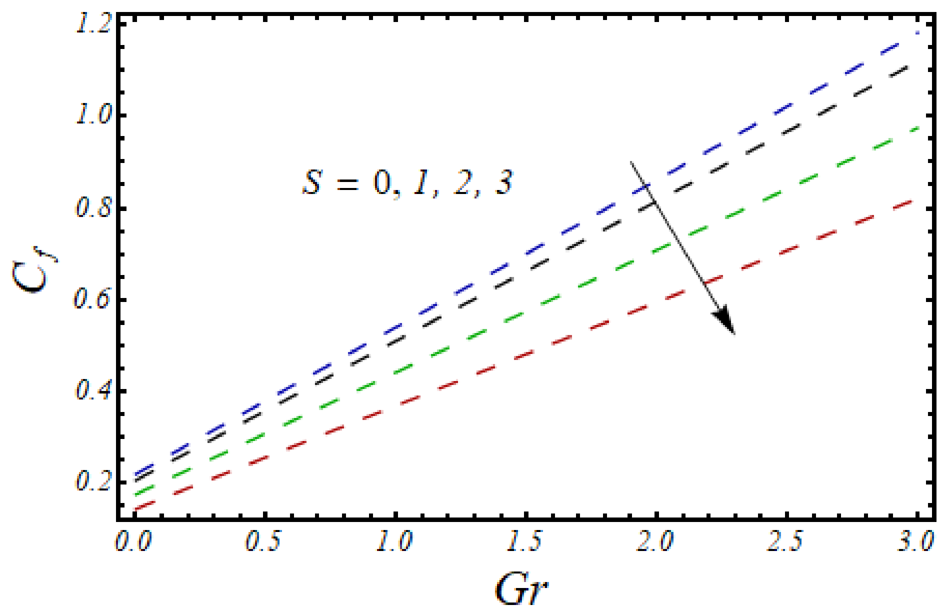

Figure 3.14: C_f versus *Gr* with *S*

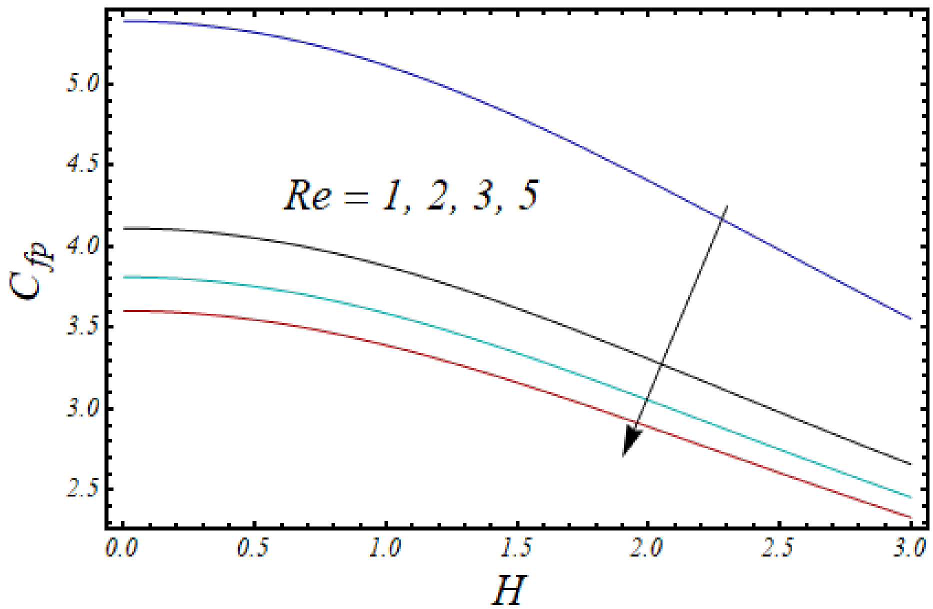

Figure 3.15: C_{fp} versus *Gr* with *Re*.

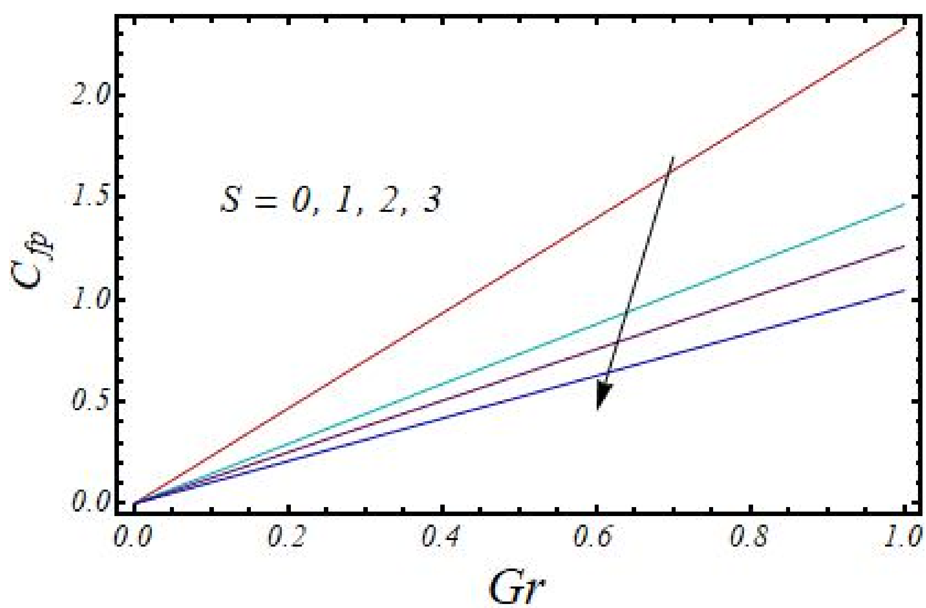

Figure 3.16: C_{fp} versus *Gr* with *S*.

Chapter 4

Darcy flow of a dusty nano fluid in a permeable channel with velocity and thermal slip condition

In this section we explored the mixed convection flow of MHD nanofluid in a vertical medium enclosed in a permeable medium with velocity and thermal slips at the walls. The channel walls are permeable. The flow of the fluid along the walls, is influenced by uniform magnetic field imposed vertically to the flow direction. The flow at the boundaries is because of the mixed convection with heat absorption/generation and suction/infusion. The model is in terms of one-dimensional PDEs. The impact of nanoparticles is determined in Fe_3O_4 and EG base fluid. The same technique, as in previous chapter, is opted to have a closed-form solution. The impact of related parameters along with the effect of velocity and thermal slip are shown by graphs. The non-dimensional skin factor and Nu are also presented graphically and discussed in detail. The current work is arranged as follows. In section 4.1 we present mathematical model for flow and heat transfer analysis. The analytical procedure is given in section 4.2. In section 4.3 we present the graphs with their results and discussion.

4.1 Problem Formulation

Here we assume a laminar, 1D, incompressible flow of a heat generating/absorbing and an electrically conducting nano fluid in a vertical channel. We consider the velocity and thermal slip at the walls. The walls of the channels are parallel to the x-axis at $y=0$ and $y=a$. B_0 is the magnetic field strength along y -axis. The nanofluid temperature at both the walls are different. It is supposed that on one plate of the channel wall $(y=0)$, the fluid is injected with velocity $(-v_o)$ and drawn off from the other plate $(y=a)$. The sematic flow of the model is presented in Figure. 4.1. Additionally, the conventional fluid EG and Fe_3O_4 nanoparticles are supposed to be in thermal state.

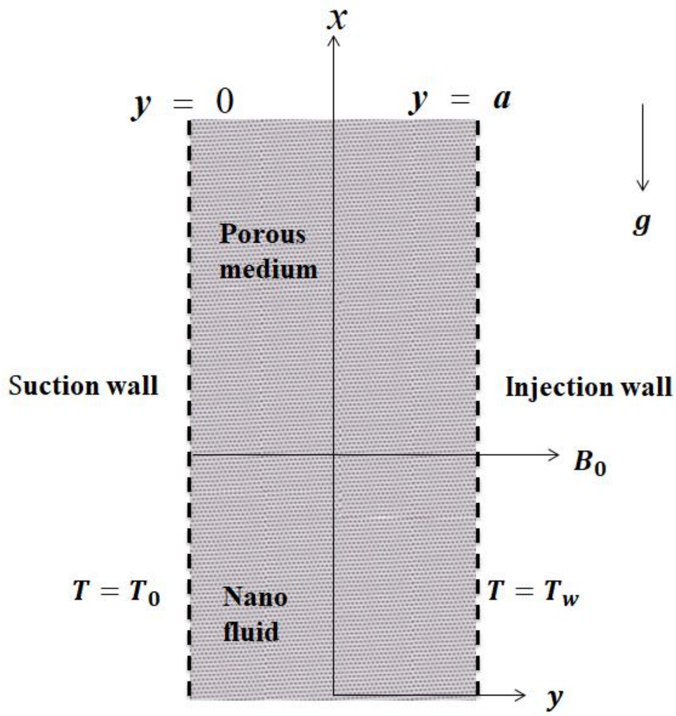

Figure.4.1: Geometry of the channel

The thermophysical properties of BF i. e. EG are mentioned in Table 4.1. The flow equations with these assumptions can be written as

$$(u)_t - v_0 (u)_y = -\frac{1}{\rho_{nf}}(P)_x + \upsilon (u)_{yy} - \frac{\upsilon}{K_{nf}} u + \frac{N_0 K_0}{\rho_{nf}}(u_p - u) - \frac{\sigma_{nf}}{\rho_{nf}} B_0^2 u + g\beta (T - T_0), \quad (4.1)$$

$$(u_p)_t = K_0 (u - u_p), \quad (4.2)$$

$$(T)_y = \frac{K_{nf}}{(\rho_{Cp})_{nf}}(T)_{yy} - \frac{1}{(\rho_{Cp})_{nf}}(q)_y \pm \frac{Q}{(\rho_{Cp})_{nf}}(T - T_0). \quad (4.3)$$

The IBCs of the problem are

$$u(y,0)=0, \quad u_p(y,0)=0, \quad T(y,0)=T_f, \quad (4.4)$$

$$u(a,t) = -l_1 (u)_y, \quad u_p(a,t) = 0, \quad T(a,t) = T_w - d_1 (T)_y, \tag{4.5}$$

$$u(0,t) = l_2 (u)_y, \quad u_p(0,t) = 0, \quad T(0,t) = T_0 + d_2 (T)_y. \tag{4.6}$$

The expressions for μ_{nf}, ρ_{nf}, α_{nf}, β_{nf}, σ_{nf}, $(\rho c_p)_{nf}$ and k_{nf} are expressed as

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$
$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}, \quad (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_s, \tag{4.7}$$
$$\beta_{nf} = (1-\phi)\beta_f + \phi\beta_s, \quad \frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\phi\left(\frac{\sigma_s}{\sigma_f} - 1\right)}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \phi\left(\frac{\sigma_s}{\sigma_f} - 1\right)}.$$

Table 4.1: Thermo-physical properties of NFs and NPs.

Liquids and nano particles	$\rho(Kg/m^3)$	$c_p(k/kg)$	$k(wK/m)$	$\beta * 10^{-5}(K^{-1})$	$\sigma(S/m)$
EG	113.2	2410	0.252	1.89	$1.07*10^{-6}$
Fe_3o_4	5200	670	6	0.5	25000

The dimensionless governing equations with suitable BCs are

$$\mathrm{Re}(u)_t = -\frac{1}{A}(P)_x + S(u)_y + \frac{1}{AB}(u)_{yy} - \left(\frac{s^2}{AB} + \frac{H^2 c}{A} + \frac{l}{A}\right)u + \frac{l}{A}u_p + Gr\,D\theta, \tag{4.8}$$

$$\mathrm{Re}\, M\,(u_p)_t = K_0(u - u_p), \tag{4.9}$$

$$F(\theta_0)_{yy} + Q_1(\theta_0)_y + m^2\theta_0 = 0. \tag{4.10}$$

After transformation the IBCs taken the form

$$u(y,0) = 0, \quad u_p(y,0) = 0, \quad T(y,0) = T_f, \tag{4.11}$$

$$u(0,t) = \gamma_1 (u)_y, \quad u_p(0,t) = 0, \quad \theta_0(0) = \alpha_1 \theta'(0), \tag{4.12}$$

$$u(1,t) = -\gamma_2 u_y, \quad u_p(1,t) = 0, \quad \theta_0(1,t) = 1 - \alpha_2 \theta'(1), \tag{4.13}$$

where $\gamma_1 = l_1$ and $\gamma_2 = l_2$ are the velocity slip parameters and $\alpha_1 = ad_1$ and $\alpha_2 = -ad_2$ are thermal slip parameters.

4.2 Analytical solution

To solve the Eqs. (4.9), (4.10) and (4.11), we use Eq. (3.15) and obtain

$$\frac{1}{AB}(u_0)_{yy} + S(u_0)_y - Qu_0 = -\frac{\lambda}{A} - Gr\,\theta_0\,D, \tag{4.14}$$

$$u_{p_0} = \frac{u_0}{1 + i\omega \mathrm{Re}\, M}, \tag{4.15}$$

$$\frac{K_{nf}}{K_f}(\theta_0)_{yy} + \Pr S\, E(\theta_0)_y + m^2\theta_0 = 0, \tag{4.16}$$

with IBCs as

$$u(\mathrm{y},0) = 0, \quad u_p(y,0) = 0, \quad \mathrm{T}(y,0) = T_f, \tag{4.17}$$

$$u(0,t) = \gamma_1 u_y, \quad u_p(0,t) = 0, \quad \theta_0(0,t) = \alpha_1\,\theta'(0), \tag{4.18}$$

$$u(1,t) = -\gamma_2\,u_y, \quad u_p(1,t) = 0, \quad \theta_0(1,t) = 1 - \alpha_2\,\theta'(1) \tag{4.19}$$

By solving Eq. (4.18) together with (4.20) and (4.21), we attained the temperature for fluid as

$$\theta = \frac{Exp\left[-\frac{Q_1(-1+y)+A_1 y}{2F}\right] + \left(e^{\frac{A_1 y}{F}} A_4 + A_5\,y - 2F - A_6\right)}{A_3}. \tag{4.20}$$

Using Eq. (4.16) together with (4.20) and (4.21), we attained the solution as

$$u = C_3\,e^{Q_9 y} + C_4\,e^{Q_{10} y} + Q_8 + Q_{11}\,e^{Q_2 y} + Q_{12}\,e^{Q_3 y}, \tag{4.21}$$

From Eq. (4.17), dust particles velocity is attained as

$$u_p = \frac{1}{1 + i\omega \mathrm{Re}\, M}\left(C_3\,e^{Q_9 y} + C_4\,e^{Q_{10} y} + Q_8 + Q_{11}\,e^{Q_2 y} + Q_{12}\,e^{Q_3 y}\right). \tag{4.22}$$

The skin friction for dusty fluid is attained as

$$C_f = C_3\,Q_9\,e^{Q_9} + C_4\,Q_{10}\,e^{Q_{10}} + Q_{11}\,Q_2\,e^{Q_2} + Q_{12}\,Q_3\,e^{Q_3}, \tag{4.23}$$

The skin friction for dusty particle is attained as

$$C_f = \frac{1}{1 + i\,\omega\,\mathrm{Re}\,M}\left(C_3\,Q_9\,e^{Q_9} + C_4\,Q_{10}\,e^{Q_{10}} + Q_{11}\,Q_2\,e^{Q_2} + Q_{12}\,Q_3\,e^{Q_3}\right), \tag{4.24}$$

$$Nu = \frac{A_7\,Exp\left[\dfrac{-Q_1(-1+y)+A_2 y}{2F}\right]}{A_3}. \tag{4.25}$$

The constants in the above expressions are set out in Appendix I.

4.3 Results and Discussion

The results for velocity, temperature, Nu and C_f are discussed and the graphs are shown in Figures. 4.2 - 4.12 so that the impact of each parameter can be seen.

From Figure 4.2 we examined the impact of ϕ on velocity of heat absorbing fluid and dust particles for the case of suction. This Figure. depicts that both the dust particles and fluid velocity are increasing with the increasing values of ϕ, however the dust particles velocity is less than the fluid velocity. This behavior of velocity is found to be similar qualitatively to the results of Hajmohammadi et al. [50], however they use cu in water and Ag as a based nanofluid. Figure. 4.3 was prepared to discuss the effects of velocity and velocity profile for heat *Q* and *S*, with the change in ϕ. The Figure shows an increment in both velocities of fluid as well as particle with a rise in ϕ. The magnitude of fluid velocity is higher than particle velocity. Moreover the parabolic flow is shifted towards the right plate of the channel. The variation in fluid and particle velocities through the channel with change in slip parameters is shown in Figure. 4.4. The increase in both velocities is noted in the Figure. However the variation in fluid velocity is more than the particle velocity.

Figure. 4.5 is graphed to show the variation of ϕ, on the fluid flow temperature in cases of *S*. with the increase in ϕ, the fluid temperature rises. when the fluid is blown into the channel the fluid temperature is higher whereas when the fluid is taken from the channel the temperature reduces.

It is examined from Figure. 4.6 that temperature graphs decreases as augmentation in suction parameter (S>0) while on escalating the values of infusion parameter (S<0) temperature outline escalates. The temperature profile on Q is showed in Figure. 4.7. we realized that $EG-Fe_3O_4$ temperature is an increasing function of Q, furthermore the thermal boundary is increased with Q. From Figure. 4.8 it is noted that an increasing in the Navier slip enlarge the flow of the suction plate owing to enhanced gas–molecule contact with the suction wall. From Figure.4.9 we see that *Nu* increases with the increase of ϕ. From Figure.4.10 as the thermal slip value increases the *Nu* also increases. The impact of ϕ of nanoparticles on non dimensional temperature is plotted in Figure. 4.11 in the presence of S parameter thermal boundary layer of $EG-Fe_3O_4$ nanofluid is increased because of various values of ϕ. From Figure. 4.12 we see that C_f reduces with the increment in ϕ .-

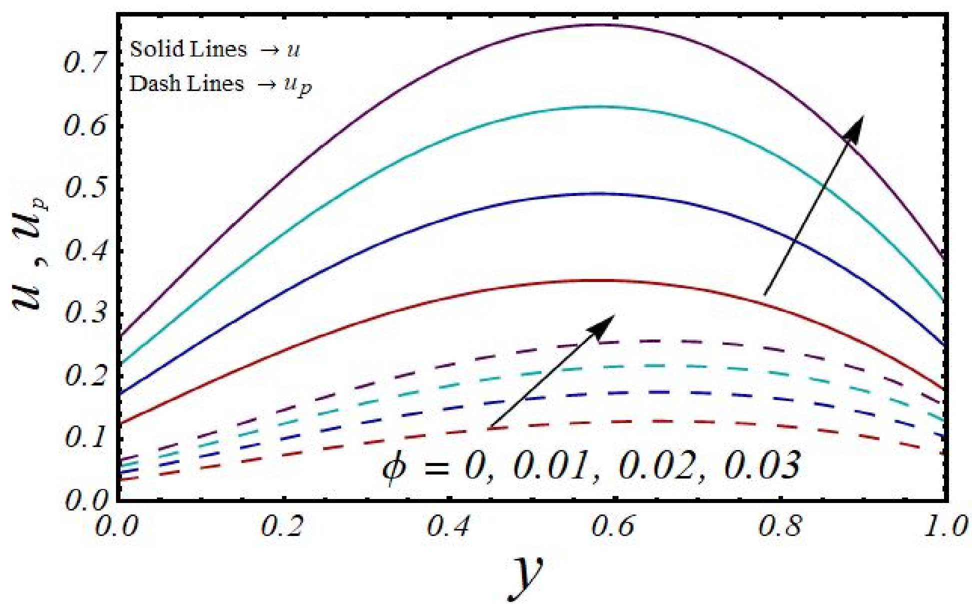

Figure 4.2: u, u_p for ϕ with $S=1, Q=2$.

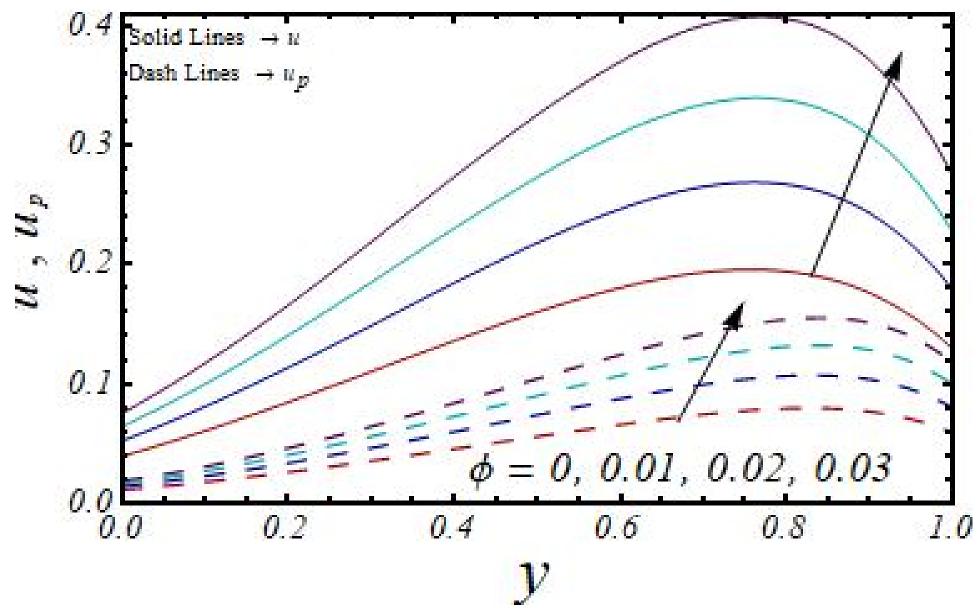

Figure 4.3: u, u_p for ϕ with $S=-1, Q=-2$.

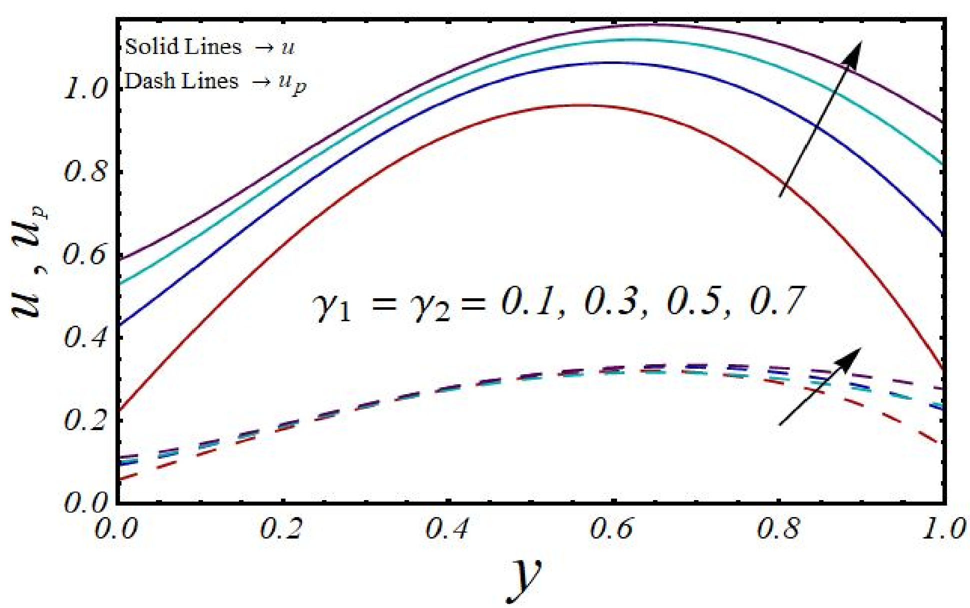

Figure 4.4: u, u_p for γ_1, γ_2 with $\phi = 0.05$, $Q=2$, $S=1$.

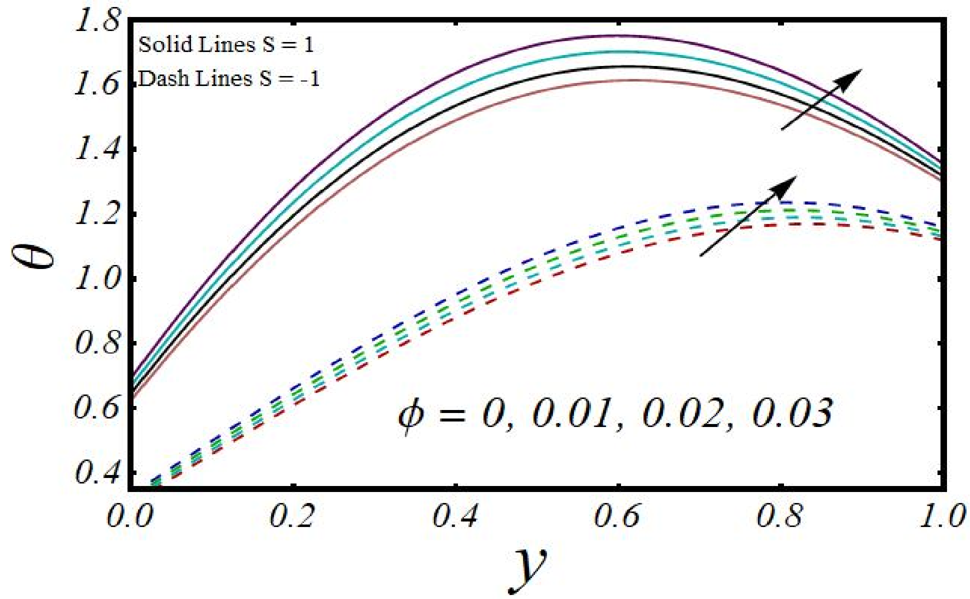

Figure 4.5: θ for ϕ

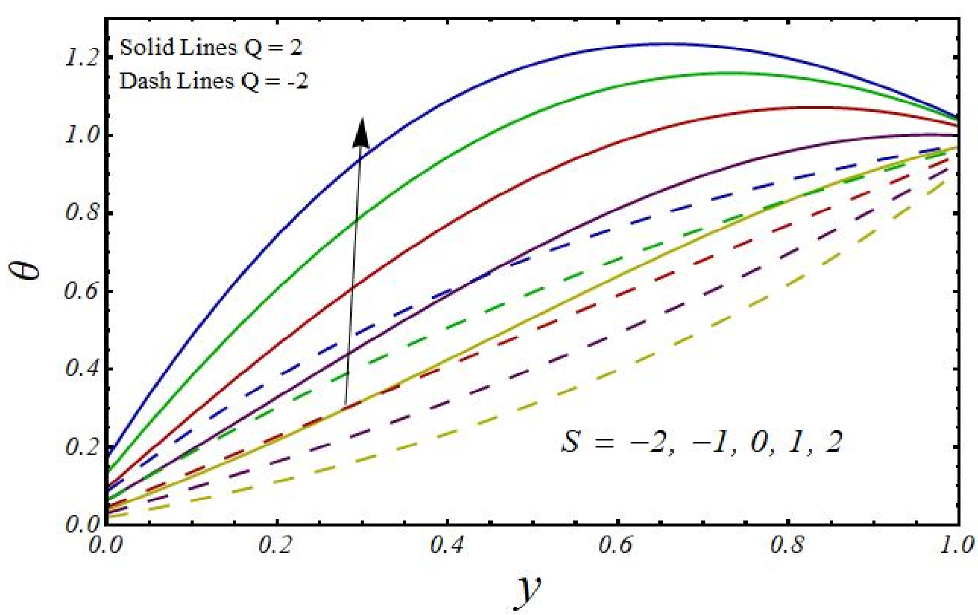

Figure 4.6: θ for S

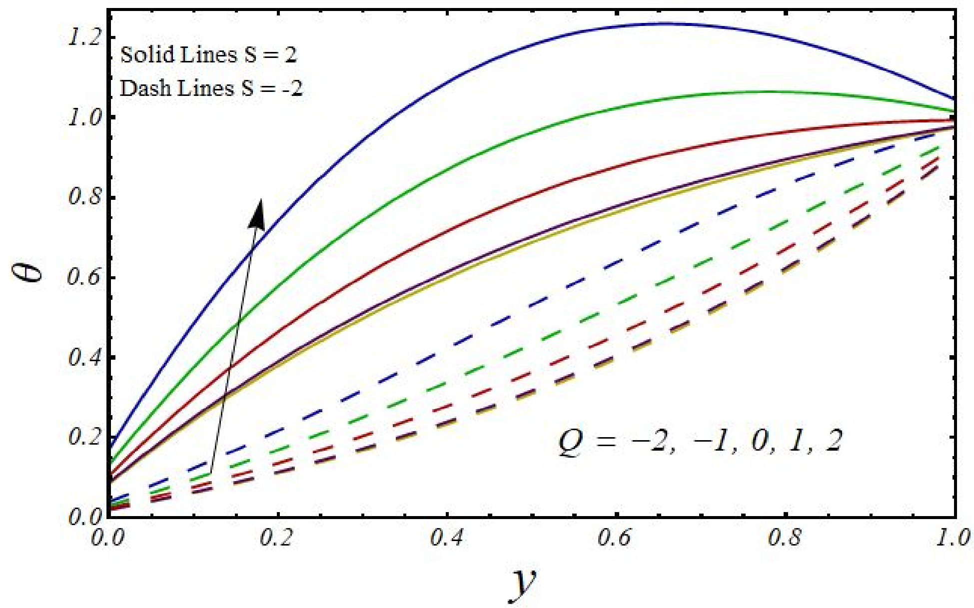

Figure 4.7: θ for Q.

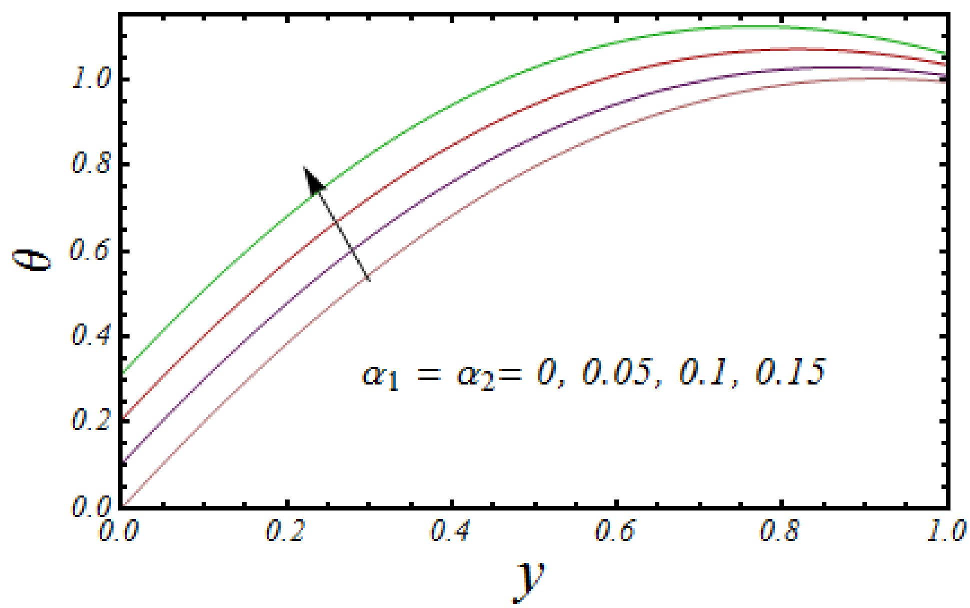

Figure 4.8: θ for α_1, α_2.

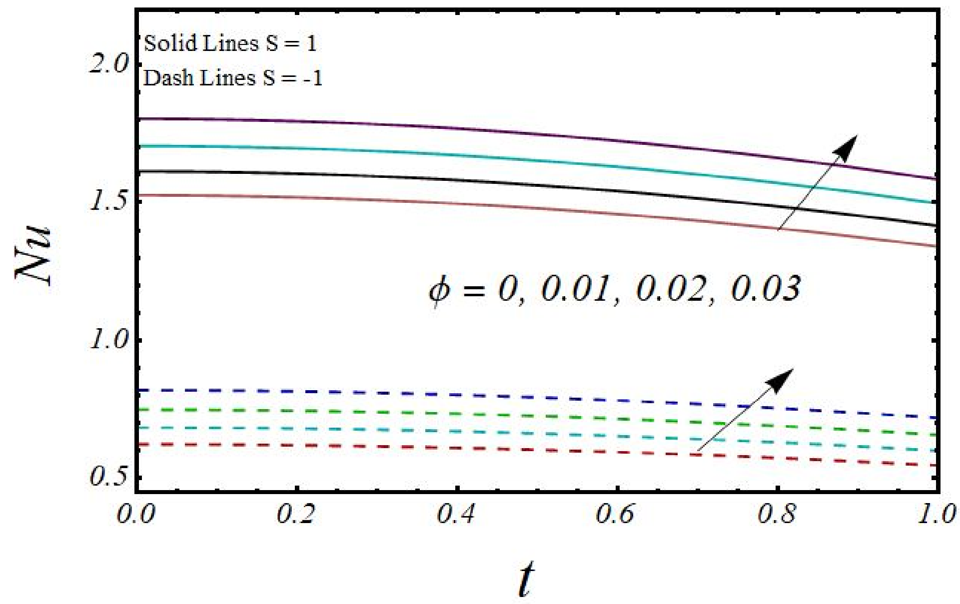

Figure 4.9: *Nu* versus t with ϕ.

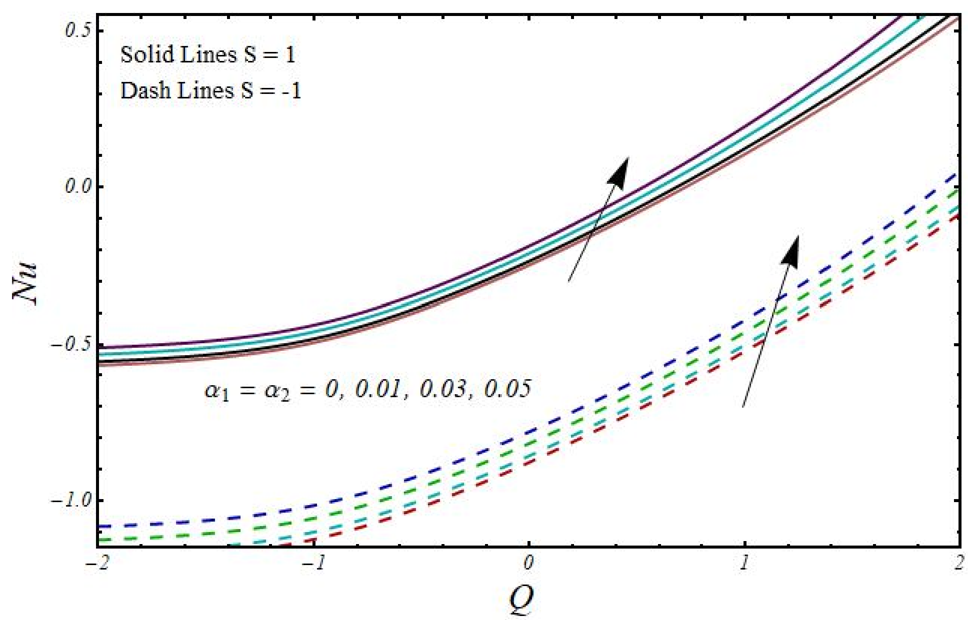

Figure 4.10: *Nu* versus Q with α_1, α_2.

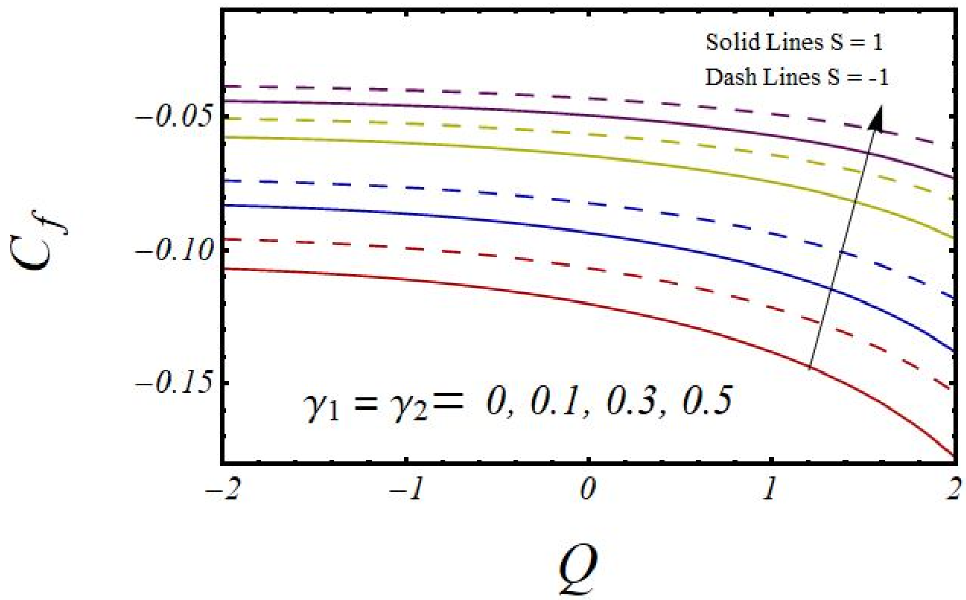

Figure 4.11: Cf versus Q with γ_1, γ_2.

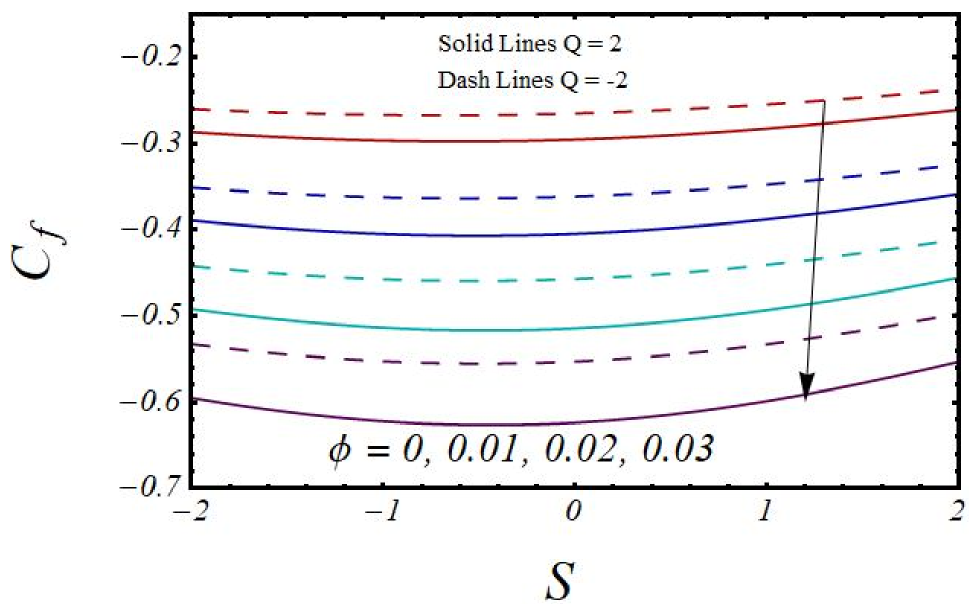

Figure 4.12: Cf versus S with ϕ.

Chapter 5

Conclusions

In this thesis two problems have been analyzed where first problem is about review paper and second problem is the extension work for it. Conclusion of both the problems are as following:

5.1 Chapter 3

In this chapter we have studied the the impacts of thermal radiation, attractive field and buoyancy force on temperature exchange to oscillatory stream of dusty fluid with penetrable medium in a vertical channel are discussed. Following conclusions are drawn:

- There is an increment in temperature of dusty fluid while there is a decrement in the Nusselt number.
- In dusty liquid and dusty particles the skin friction parameter decreases with s and H where increases with Gr and N .
- The decreasing effect of H and s in particles speed will increase the effect of N and Gr in dusty fluid and dust particles .

The outcomes achieved will be helpful in recovery of raw petroleum, waste water treatment and so forth.

5.2 Chapter 4

In this dissertation we have investigated the dusty fluid flow and heat exchange over a vertical channel. we have analytically discussed heat exchange in combined convection flow of nanofluids in a vertical medium in the presence of suction/infusion and heat generation/absorption effects and also in the presence of velocity and thermal slips. The

EG is considered as a conventional base fluid with dispersion of Fe_3O_4 nanoparticles. Formulation of velocity and temperature are attained. The dispersion of nano sized particles Fe_3O_4 in EG based nanofluid has higher thermal conductivity and viscosity. Furthermore the time independent fluid flow through a vertical medium is explored. The impact of slips on the channel walls are also discussed. SOV method is used to solve the governing equations. To demonstrate the impacts of governing parameters on fluid velocity and temperature.0 results are plotted in the graphs and the conclusions drawn are as follows.

- Fluid velocity increases as we increase the values of the slip parameters γ_1 and γ_2 whereas the fluid velocity decreases with an increase in nanofluid volume fraction φ. Based on numerical experiments, it is concluded that both the velocity and temperature profiles are significantly affected by φ.
- The size of nanoparticles start influencing at a specific limit of ϕ .
- The increment in the temperature slip results in increment of the fluid temperature whereas the heat exchange decreases at the suction wall.
- It is seen that there is a specific limit of increasing size of nanoparticles upto 30nm which results in increasing the velocity and after that for more increasing size of nanoparticles there is no change to observed
- Due to viscosity, it was noted that ϕ increases when the velocity of nanofluids increases.
- The velocity increases as Q rises.
- Increasing the slip parameter values improves the fluid's velocity, while reducing the fluid velocity, ϕ rises.
- We noted that both the temperature and velocity profiles are effected by ϕ .
- Furthermore, the magnetic parameter detains the nanofluid motion whereas porosity accelerates it.
- In case of suction EG -based and Fe_3o_4-based nanofluid have a higher magnitude of velocity as compared to the injections case.

- Moreover, the magnetic parameter retards the nanofluid motion whereas porosity accelerates it. Each EG -based and Fe_3o_4-based nanofluid in the suction case have a higher magnitude of velocity as compared to the injections case.
- At the end, we noted that due to suction/infusion various kinds of nanoparticles have various impacts on the temperature and velocity.

The results achieved are important in the cooling of electronic devices.

Appendix

$$A = (1-\phi)+\phi\frac{\rho_s}{\rho_f},\quad B=(1-\phi)^{2.5},\ C = 1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi},$$

$$D = (1-\phi)+\phi\frac{\beta_s}{\beta_f},\ \ E = (1-\phi)+\phi\frac{\rho_s\, C\,\rho_s}{\rho_f\, C\,\rho_f},\quad F = \frac{\mathrm{k}_{nf}}{\mathrm{k}_f};$$

$$q = \frac{s\,s}{AB}+\frac{H\,C}{A}+\imath\,\omega\,R+\frac{l}{A}\left(\frac{\imath\,\omega\,R\,M}{1+\imath\,\omega\,R\,M}\right),\ C_1 = \frac{Q_5}{Q_5\,Q_6-Q_4\,Q_7},$$

$$C_2 = \frac{Q_4}{-Q_5\,Q_6-Q_4\,Q_7},\ \ C_3 = -\frac{-Q_{15}\,Q_{17}+Q_{14}\,Q_{18}}{Q_{14}\,Q_{16}-Q_{13}\,Q_{17}},\ \ C_4 = -\frac{-Q_{15}\,Q_{16}-Q_{13}\,Q_{18}}{Q_{14}\,Q_{16}-Q_{13}\,Q_{17}},$$

$$Q_1 = \Pr S\,E,\ \ Q_2 = \frac{-Q1-\sqrt{-4\,F\,m^2+Q1^2}}{2F},$$

$$Q_3 = \frac{-Q1+\sqrt{-4\,F\,m^2+Q1^2}}{2F},\ \ Q_4 = 1-Q_2\,\alpha_1,\ \ Q_5 = 1-Q_3\,\alpha_1,$$

$$Q_6 = e^{Q2}(1+Q_2\,\alpha_2),\ \ Q_7 = e^{Q3}(1+Q_3\,\alpha_2),\ \ Q_8 = \frac{\lambda}{Aq^2},$$

$$Q_9 = \frac{-AS-\sqrt{A}\sqrt{4Bq^2+AS^2}}{2B},\ \ Q_{10} = \frac{-AS+\sqrt{A}\sqrt{4Bq^2+AS^2}}{2B},$$

$$Q_{11} = \frac{AC_2\,DD\,e^{Q_2 y}\,Gr}{Aq^2-BQ_2^2-AQ_2\,S},\ \ Q_{12} = \frac{AC_2\,DD\,e^{Q_3 y}\,Gr}{Aq^2-BQ_3^2-AQ_3\,S},$$

$$Q_{13} = 1-Q_9\,\gamma_1,\ \ Q_{14} = 1-Q_{10}\,\gamma_1,\ \ Q_{15} = Q_8+Q_{11}+Q_{12}-(\mathrm{Q}_{11}\,\mathrm{Q}_2+\mathrm{Q}_{12}\mathrm{Q}_3)\,\gamma_1,$$

$$Q_{16} = (1+\mathrm{Q}_9\,\gamma_2)\,e^{Q_9},\ \ Q_{17} = (1+\mathrm{Q}_{10}\,\gamma_2)\,e^{Q_{10}},$$

$$Q_{18} = Q_8+Q_{11}\,e^{Q_2}+Q_{12}e^{Q_3}+(\mathrm{Q}_{11}\,\mathrm{Q}_2\,e^{Q_2}+\mathrm{Q}_{12}\mathrm{Q}_3\,e^{Q_3})\,\gamma_2,$$

$$m = \sqrt{N^2+Q-i\,\omega\,\mathrm{Re}\Pr EE},\quad \textit{and}\quad q = \frac{s^2}{AB}+\frac{H^2C}{A}+i\,\omega\,\mathrm{Re}+\frac{l}{A}\left(\frac{i\,\omega\,\mathrm{Re}\,M}{1+i\,\omega\,\mathrm{Re}\,M}\right)$$

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