Analysis of nanofluid flow with unequal diffusivities of H-H reactions over a stretching/shrinking cylinder

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Dedicated To

I am dedicating this thesis to my beloved parents ***Mr. Gohar Rehman*** *and* ***Mrs. Naseem Akhtar*** *who have meant everything to me, thank for your endless love, pray, support, sacrifice, guidance, patience and everything you have done since I was born.*

My beloved maternal grand father ***Mr. Ayub*** *(late), thank for your pray, care, advice, guidance and support. His memories continue to regulate my life. May Almighty Allah grant you highest rank in Jannah. Ameen.*

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In the name of Allah the most Merciful and Beneficent

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Abstract

The objective is to investigate the outcomes of unequal diffusivities of homogeneous-heterogeneous (H-H) reactions on the nanofluid flowing over a shrinking cylinder. The cylinder is exposed to a perpendicular driven magnetic field. The generalized slip effects will also be studied. Copper-water nanofluid is subject to magnetic field and generalized slip boundary effects in this problem. With the assistance of similarity transformations, the ensuring partial differential equations are modified in dimensionless ordinary differential equation and by the use of shooting technique the resulting numerical solutions will be obtained. The physical effect of all fluid parameters on flow arenas are presented graphically and described in detail with tables. Dual solution is found to exist for shrinking cylinders, while the solution for stretching cylinders is unique. Dual solution is found to exist for shrinking cylinders, while the solution for stretching cylinders is unique. The comparison of the present results with the previous results is presented and found to be in good agreement.

Nomenclature

z, r	cylindrical polar coordinate
B_0	uniform magnetic field
p	pressure
u, w	velocity components
a_0, b	Constant
r, z	cartesian coordinate
N	velocity slip length
N_1	initial value of velocity slip parameter
δ	ratio of mass diffusion parameter
T_∞	ambient temperature of fluid
T_w	temperature at cylindrical surface
$\mathbf{L}$	gradient of the velocity
d/dt	material derivative
T	nanofluid temperature
S	unsteadiness parameter
U	constant mass transfer/suction
K	parameter of homogeneous reaction
q	heat flux
K_p	parameter of heterogeneous reaction
$\Pr$	prandtl number
D_A , D_B	Diffusion coefficients
p	nanofluid pressure
Nu	Nusselt number
E	Electric field
Sc	Schmit number
J	current density

a, b	concentration of chemicals a and b
k	thermal conductivity
g	gravitational acceleration
D_B	Brownian diffusion coefficient
C_p	specific heat and constant temperature
C_f	skin friction
C	nanoparticles connection
k_n	reaction rate of the homogeneous chemical response of the nth-order.
u_t	tangential sheet velocity
Greek letters	
γ	mass suction parameter
σ	electrical conductivity
τ_w	surface shear stress
$\rho\mathbf{b}$	body force per unit mass
τ	Cauchy stress tensor
λ	velocity slip parameter
δ	variable viscosity coefficient
η	dimensionless variable
θ	dimensionless temperature
ρ	nano fluid density
μ	dynamic viscosity
υ	kinematic viscosity coefficient
ε	stretching/shrinking cylinder
$\alpha(t), \beta(x,t)$	Constants
δ	ratio of mass diffusion parameter
α	slip parameter
$\alpha *$	the steady slip length of Navier
ρ_f	density for copper water fluid
ρ_s	density for base fluid

$\left(C_p\right)_f$	specific heat parameter
κ_f	thermal conductivity
$\left(C_p\right)_s$	specific heat parameter
κ_s	thermal conductivity for base fluid
ρ_{nf}	effective density
$\left(\rho C_p\right)_{nf}$	heat capacitance
κ_{nf}	thermal conductivity
φ	nanoparticle volume fraction
$\beta *$	the reciprocal critical shear rate

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Chapter 1

Introduction

This chapter presents few fundamental definitions of the flow phenomena and associated equations like continuity, momentum etc. Additionally, few fundamental solution's strategies are presented such as Runge-Kutta and shooting technique.

1.1 Basic definitions

1.1.1 Fluid

A continuous matter that has a tendency to flow and adapt to the outline of the container (liquid or gas) is called fluid. Fluids are further classified as ideal fluids (no resistance to flow), real fluids (some resistance to flow). Flow depends upon the implicit properties of the matter itself that include viscosity, compressibility and thickness. Examples are plasma motions in stars, air flow across the aero plane wings, liquid streaming through a pipe or capillary etc.

1.1.2 Nano fluid

Nano fluid is a fluid comprising nanometer-sized suspension (100 mm) like oxides, carbides or nitrides in a base liquid such as ethylene, glycol or water etc. Nanofluids have distinctive chemical and physical and chemical characteristics, also higher thermal conductivity than regular fluids or we can say that these nanoparticles boost the conductivity process. Examples of the certain nanofluid applications can be seen in nano electromechanical systems, sophisticated advanced power conversion devices such as heat pipe and solar collectors, cancer therapy, microchip cooling, imaging and sensing. Recently most of the investigations of heat flow over a cylinder were concentrated by research workers and its applications can be easily seen in various engineering branches like in wire drawing, fiber glass production, manufacturing of plastics, hot rolling etc.

1.1.3 Heat Transfer

It is usually a renowned idea that heat is exchanged from one body to other by conduction (solids), convection (liquids) and radiation (all the things which able to radiate). Heat has the property that it always transfers from a higher to lower temperature the more the temperature the larger heat will be emitted by a body. When two objects are placed in one another the cooler body will keep on obtaining heat from the hotter body till it reaches the same temperature. The research of thermal generation / absorption is defined in many physical circumstances such as endothermic or exothermic chemical reactions. The inclusive applications are like drawing of plastic sheets, continuous casting, food stuff processing exchangers instruments, fiber and wire coating etc. Thus, proves that heat transfer engrossed a significant consideration in science and technology.

1.1.4 Magnetohydrodynamics

Magnetohydrodynamics (MHD) is basically linked to fluid dynamics with excellent electrical conduction. It is particularly related to the effects that take place after the interaction between fluid motion and attractive field. This type of field is generated through electric current induced internally or externally in the fluid.

1.1.5 Slip flow

In many cases, (no-slip) boundary condition is established whereas different practical technological applications reveal flow regimes in which fluid speed does not collide with solid boundary velocity and fluid slippage occurs at solid boundaries. In the present era, the slip flow scheme is widely studied and scientists focused on micro-scale evaluation in (MEMS) associated with the velocity slip embodiment.

1.1.6 Chemical reaction

The process in which the starting material differs from the product after chemi cal change is referred to as chemical reaction.

For example

$$Fe+O_2=FeO_2.$$

Examples are from our daily life like photosynthesis, curdling of milk and rusting of iron

1.1.7 Classification of reactions

i) Heterogeneous reactions

A chemical reaction in which reactants are multi-phase elements (such as solid and gas, solid and fluid) or we can say that one or two chemical reactants undergo chemical change in a user interface (such as a strong catalyst) is called a heterogeneous reaction. Example of heterogeneous are electrochemical reactions that occur in batteries and cells, metal acids, metal acid responses are the most significant illustration of heterogeneous reactions. The heterogeneous reactions are basically linked to catalysis.

ii) Homogeneous reactions

A chemical reaction in which reactants are single phase elements such as liquid, solid and gas is called homogeneous reactions. The most common example is reaction of house hold gas with oxygen to produce the flame, formation and dispersion of fog, cooling towers, temperature distribution etc. are some demonstrative arenas which play important part in mass and heat removal with chemical reaction. Comparatively homogeneous reactions are simpler as they are dependent on the nature of substance.

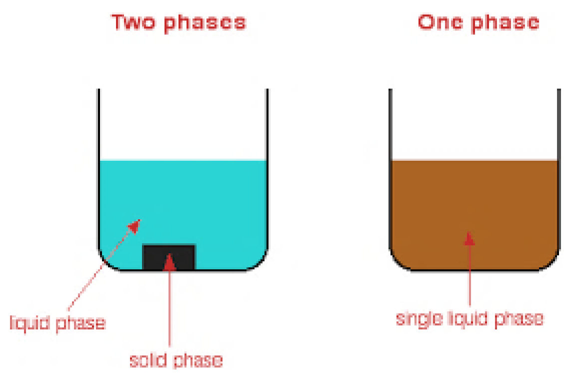

Figure 1.1 The physical reaction mechanism

1.1.8 Mass transfer

The movement of one particle from a greater concentration region to a decreas ed concent-ration region is referred to as mass exchange. Eventually mass exchange is the total system transport having more than two parts the concentration of which vary from point to point. Basically it is done to reduce the concentration difference. Examples are purification of blood in kidney, fragrance of perfume in surroundings, water evaporation from pond to ground.

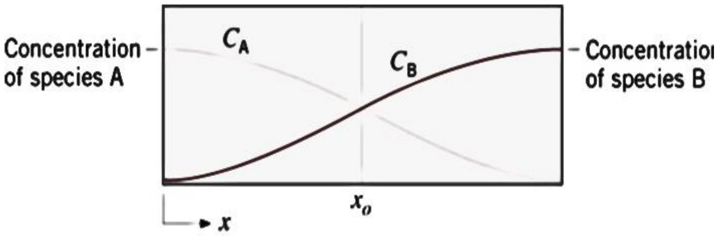

Figure 1.2 Mass transmission structure

1.2 Basic equations

The basic equations are described as

(a) The continuity equation for incompressible fluid

$$\text{div}\,(\mathbf{V}) = 0. \tag{1.1}$$

(b) The momentum equation

$$\rho\frac{d(\mathbf{V})}{dt} = \text{div}\boldsymbol{\tau} + \rho\mathbf{b}. \tag{1.2}$$

(c) The equation of energy

$$\rho(c_p)\frac{d(T)}{dt} = \boldsymbol{\tau}\cdot\mathbf{L} - \text{div}\,\mathbf{q}. \tag{1.3}$$

(d) The concentration equation

$$\frac{d\boldsymbol{C}}{dt} = D\nabla^2 C + k_n C^n \tag{1.4}$$

1.3 Solution methods

Various physical and engineering problems are structured as partial / ordinary differential equations (ODEs) that are not linear. Similarity transformations are used to change partial differential equations (PDEs) into normal differential equations and are further resolved by various numerical methods as well as analytical methods. Fluid flow problems are manipulated with the help of different techniques of numerical e.g. finite difference method, shooting method, Runge-Kutta method, F.E.M.

1.3.1 Runge-Kutta Method

The Runge-Kutta method is the strong predictor to solve numerical alternative for differential equations. A method in which ordinary differential equations are integrated numerically.

$$\frac{d^2}{dx^2}(y) = f\left(x, y, \frac{d}{dx}(y)\right), \tag{1.5}$$

with IC's

$$y_0 = y(x_0),\ \dot{a}. = \frac{d}{dx}\left(y(x_0)\right) \tag{1.6}$$

The second order original valueproblemmentionedabove is transformed by defining The first order scheme into the original value problem

$$g(x, y, z) = \mathrm{z} = \frac{d}{dx}(y), \tag{1.7}$$

thus, we get

$$f(x, y, z) = \frac{d}{dx}(z), \tag{1.8}$$

and the ICs are

$$y_o = y(x_o),\ \dot{a}. = z(x_o) \tag{1.9}$$

The R-K method for the differential Eqs of first order scheme described above. (1.7) and (1.8) are referred to as

$$y_{n+1} = y_n + \frac{1}{6}(c_1 + c_2 + c_3 + c_4), \tag{1.10}$$

and

$$z_{n+1} = z_n + \frac{1}{6}(d_1 + d_2 + d_3 + d_4), \tag{1.11}$$

where

$$c_1 = gh(x_n, y_n, z_n), \qquad d_1 = fh(x_n, y_n, z_n),$$

$$c_2 = gh\left(x_n + \frac{h}{2}, y_n + \frac{c_1}{2}, z_n + \frac{d_1}{2}\right), \qquad d_2 = fh\left(x_n + \frac{h}{2}, y_n + \frac{c_1}{2}, z_n + \frac{d_1}{2}\right),$$

$$c_3 = gh\left(x_n + \frac{h}{2}, y_n + \frac{c_2}{2}, z_n + \frac{d_2}{2}\right), \qquad d_3 = fh\left(x_n + \frac{h}{2}, y_n + \frac{c_2}{2}, z_n + \frac{d_2}{2}\right),$$

$$c_4 = gh(x_n + h, y_n + c_3, z_n + d_3), \qquad d_4 = fh(x_n + h, y_n + c_3, z_n + d_3),$$

The uniform step size h is denoted as

$$h = \frac{x_{n_0} - x_0}{n_0}. \tag{1.12}$$

In the above mentioned Eq. (1.12) n_0, x_0, x_n are representing number of steps, initial and last value

1.3.2 Shooting method

Shooting technique isbasically an iterative method in numerical analysis to resolve the problem of boundary value by converting it to the common differential equations. In
this technique we take the problemof boundary value and discover the answer at one end, then shoot the original value solver at the other end, unless the boundary condition converges to its precise value at the other end. Suppose boundary value problem for third order is defined as

$$\frac{d^3}{d\xi^3}(h) = \Phi\left(\xi, \mathrm{h}, \frac{d}{d\xi}(h), \frac{d^2}{d\xi^2}(h)\right), \tag{1.13}$$

with the BC's

$$h(0) = \hat{\gamma}, \quad h'(0) = 0, \quad h'(l) = A. \tag{1.14}$$

where domain of solution is $\xi = 0$ to $\xi = l$, and unknown IC's is selected as

$$\frac{d^2}{dh^2}(h(0)) = \underline{s} \tag{1.15}$$

Eq. 1.13 is reducing as

$$\frac{d}{d\xi}(h) = v, \tag{1.16}$$

$$\frac{d}{d\xi}(v) = w, \tag{1.17}$$

$$\frac{dw}{d\xi} = \Phi(\xi, \mathrm{h}, v, w), \tag{1.18}$$

and the IC's convert into

$$h(0) = \hat{\gamma}, \quad v(0) = 0, \quad w(0) = \underline{s}. \tag{1.19}$$

Differentiating Eqs. (1.18) and (1.19) w.r.t to $\underline{s}$

$$\frac{d}{d\xi}(w) = \frac{\partial}{\partial v}(\Phi) \times V + \frac{\partial}{\partial w}(\Phi) \times W + \frac{\partial}{\partial h}(\Phi) \times H, \tag{1.20}$$

where

$$V = \frac{\partial}{\partial \underline{s}}(v), W = \frac{\partial}{\partial \underline{s}}(w), H = \frac{\partial}{\partial \underline{s}}(h) \tag{1.21}$$

and

$$V(0)=0, \quad W(0)=0, \quad H(0)=1. \tag{1.22}$$

For systems of IVP as Eq. (1.18) and Eq. (1.19) with Eqs. 1.20 & 1.22. The supposed value of $\underline{s}$ must satisfy the condition $h'(l)=\widehat{A}$. Suppose that $v(\xi,\underline{s})$, $w(\xi,\underline{s})$ known as the solutions of IVP, the value of s_1 such that

$$h(\xi,\underline{s})-\widehat{A}=0=\tau(\underline{s}), \quad \text{(say)} \tag{1.23}$$

The Newton method can be used to improve the value of $\underline{s}$

$$\underline{s}^{(n+1)}=\underline{s}^{(n)}-\frac{\tau_1\left(\underline{s}^{(n)}\right)}{\frac{d}{d\underline{s}}\left((\tau_1)\left(\underline{s}^{(n)}\right)\right)}. \tag{1.24}$$

By substituting the value from Eq. (1.23) in Eq. (1.24), of $\tau_1(\underline{s})$, we obtain

$$\underline{s}^{(n+1)}=\underline{s}^{(n)}-\frac{\left(h\left(l,\underline{s}^{(n)}\right)-\widehat{A}\right)}{\frac{\partial}{\partial\underline{s}}\left(h\left(l,\underline{s}^{(n)}\right)\right)}. \tag{1.25}$$

Eqs. (1.16) – (1.18) is solved as follows, subject to original condition (1.19).

(i) $\underline{s}$ is selected for the missing I.C in Eq. (1.15) and termed as $\underline{s}$

(ii) Eqs. (1.16) – (1.18) is integrated for range 0 to l. with the I.C's (1.19)

(iii) Eq. (1.20) with an initial condition (1.22) is solved as an IVP from 0 to l

(iv) By using the values of $h\left(l,\underline{s}^{(1)}\right)$ and $H\left(l,\underline{s}^{(1)}\right)$ into Eq. (1.25) we get

$$\underline{s}^{(2)}=\underline{s}^{(1)}-\frac{\left[h\left(l,\underline{s}^{(1)}\right)-\widehat{A}\right]}{H\left(l,\underline{s}^{(1)}\right)}.$$

(v) All the above steps are keep on repeating until the solutions meets.

Chapter 2

Literature Review

1.1 Overview

In this chapter we represented the literature study related to, mass and heat transfer, MHD and slip flow. The fundamental concepts of techniques for solving the governing equations are also presented.

1.2 Related work

Recently, most of heat flow study over a stretching cylinder was focused by research employees owing to its importance in the field of studies. Nano fluids first identified by Choi [1] are the fluids containing the solid nanometer size distribution of metal. Nanotechnology has opened the doors for medical and engineering sciences to take the advantage such as cleaning oil, medicine diffusion in blood, cooling of electronic equipment's etc. Makinde and Aziz [2] analyzed the MHD flow nano fluid over a surface and concluded that transfer of heat is better in Cu-water than rest of the mixtures. Parvin et al. [3] studied the steady model of laminar flow in nanofluid (alumina) numerically. Moghari et al. [4] also worked on enhancement of heat transfer by using mixed convected nanofluid. Das et al. [5] investigated the nanofluid with mixed convective MHD vertical channel flow. Hayat et al.[6] evaluated the impacts of unidirectional nano fluid flow along with H-H reactions and slip impacts. Fakour et al. [7] also examined fluid flow in a channel that is subject to nanofluid in the existence of a magnetic field.

Recently most of the study of heat flow over a shrinking cylinder were concentrated by research workers and its applications can be easily seen in various engineering branches like in drawing of wire, fiber glass production, plastic manufacturing, hot rolling etc. Ishak et al. [8] evaluated the numerical solution outside the stretching cylinder for flow heat and transport.

The convective flow in a stretching cylinder was also researched by Mukhopadhay and Ishak[9]Acharya et al.[10] also studied unstable stretching cylinder with MHD boundary flow with non- uniform heat source. The analysis of H-H reactions in Sisko liquid over a stretching cylinder was explored by Malik et al. [11]. Shojaei et al.[12]examines the second grade fluid flow with Soret and Dufour i mpacts along a stretching cylinder. Heat transfer captivated a significant deliberation in various grounds of technology and science. Abel and Veena[13] also worked on the transfer of heat to porous medium. Prasad et al.[14] also explored the transfer of heat through a non-iso thermal sheet in visco-elastic liquid flow. Putley [15] also discussed the thermal imaging system in filed of medical thermology. Zaimi et al.[16] looked at unsteady flow as a result of contracting a nanofluid cylinder. Elbashbeshy et al.[17] also worked with suction and injection into a porous medium to transfer heat through the stretching surface. Magyari and Pantokratoras [18] are also investigating the impacts of boundary layer flows and effects of thermal radiation .

Abbas et al. [19] analyzes heat transfer and constant momentum properties of MHD viscous fluid near stagnation point past a stretching sheet. Hayat et al.[20] are also examining the heat transfer analysis and MHD flow features with speed and thermal slip. Fang et al.[21] also dealt with the MHD viscous fluid with a slip over a stretch sheet and found the exact solution. In a vertical channel partly filled with transparent fluid and partly fluid-saturated porous medium, Chauhan and Agrawal [22] are also studying MHD mixed convection flow. Sivaraj et al.[23] recently examined and obtained the exact solution of the MHD mixed convective flow of viscous and viscoelastic fluids in a porous medium.

Clearly, if there is no slip flow, the relative velocity is zero, whereas the relative velocity is non-zero with slip flow. Wang [24] analyzes the slip impacts of viscous flow over stretching sheet with suction. Mansur and Ishaq[25] also investigated heat slip impacts on MHD nanofluid over a shrinking/ stretching sheet. Thompson and Troian are proposing the concept of generalized slip boundary[26]. Aziz[27] evaluated the impact of both steady heat flux and slip over a plate of boundary layer flow. Mahapatra[28] shows the effect of heat absorption / generation speed slip and thermal transfer over a stretch / shrink sheet.

Formation and dispersal of fog, cooling towers, temperature distribution etc. are some demonstrative arenas which play important part in mass and heat removal with chemical reaction. Hayat et al [29]examine the boundary layer flow of carbon nanotubes across a curved surface. Heat absorption / generation in MHD stream of vertical permeable channel with chemical reactions also illustrated by Chamka [30]. Kameswaran et al.[31] examined the impact on a stretching sheet of CuH_2O and AgH_2O nanofluid of H-H reactions in a porous medium.

Khan and Pop [32] evaluate the effects of H-H reactions because of stretching sheet by applying an implicit finite difference method on a viscoelastic fluid. Chaudhary and Merkin [33] assumed first-order kinetics as the heterogeneous (surface) reaction and they demonstrated isothermal cubic kinetics as the homogeneous (bulk) reaction. Bachok et al. [34] also discussed the reactions of H-H reactions and its effects on stagnation flow point on stretching sheet. Shaw et al. [35] also worked on chemical reactions in micropolar fluids. Sheikh et al. [36] also examined the H-H reaction
ns in the presence of Casson fluid over an extending wall.

Barcelo et al. [37] evaluate the ability of mass transmission in raceway reactors. Huang et al. [38] examine the impacts of mass and heat transfer of quasi counter flow parallel plate with cooling tubes. Radiation and absorption of mass transfer analysis for MHD is also examined by Prasad et al. [39]. The fact of mass transfer flow and measurement separation behavior in elbow of $90^{\circ}$is explored by Ikrashi [40]. Heat transfer analyzes were investigated by Abbass et al.[41] due to a non-steady stretching cylinder with a partial slip condition.

Chapter 3

Analysis of partial slip boundary condition on the flow due to unsteady stretching/shrinking cylinder with suction and heat transfer

This chapter examines the evaluation of heat transfer over porous shrinking / stretching cylinder with partial slip condition. The energy and a flow equations are compared with the available information. The momentum equations are transformed into a set of nonlinear ODEs through similarity transformations and numerically solved by the R.K method. The effect of the multiple parameters concerned on the velocity and temperature stream distribution, which is further elaborated through tables and graphs. During the research, the distinctive solution for stretching cylinder and dual solutions for shrinking cylinder is examined. This chapter is basically a review of paper by Abbas et al. [41].

3.1 Problem development

We consider the t unsteady and two-dimensional flow of a viscous fluid through a permeable shrinking/stretching cylinder. Here the cylinder axis is along z -axis and along r-axis is the radial direction, as shown in Figure 3.1.The diameter of cylinder depends on time with radius $a(t) = a_0\sqrt{1-\beta t}$. Assuming the temperature at the surface of cylinder as a function of time $T_w(\mathrm{z,t})$ and where $T_w > T_\infty$ corresponds to assisting flow. The flow is axisymmetric and incompressible. The Navier-Stokes and the energy equations of the physical model.

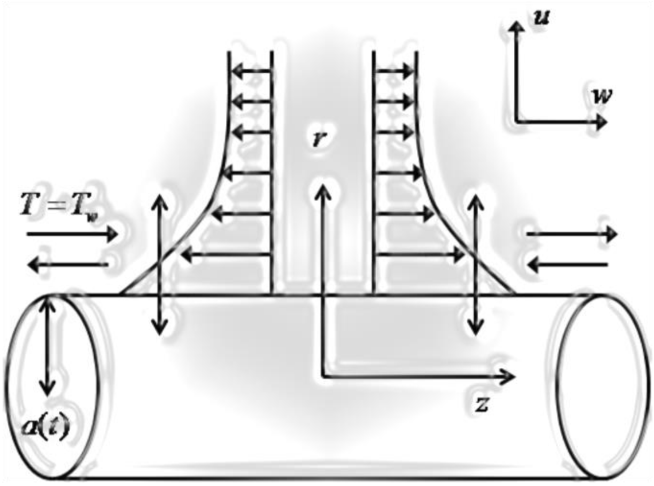

Figure 3.1 Coordinate system physical modeling

$$\frac{1}{r}(ru)_r + w_z = 0 \tag{3.1}$$

$$u_t + uu_r + wu_z = -\frac{1}{p} p_r + v\left(u_{rr} + \frac{1}{r} u_r + u_{zz} - \frac{u}{r^2} \right) \tag{3.2}$$

$$w_t + uw_r + ww_z = -\frac{1}{p} p_z + v\left(w_{rr} + \frac{1}{r} w_r + w_{zz} \right) \tag{3.3}$$

$$\rho c_p \left(T_t + uT_r + wT_z \right) = k\left(T_{rr} + \frac{1}{r} T_r + T_{zz} \right) \tag{3.4}$$

The appropriate BCs are

$$u = \frac{U}{\sqrt{1-\beta t}}, w = \varepsilon \frac{1}{a_0^{\ 2}} \frac{4vz}{1-\beta t} + N\mu \frac{\partial w}{\partial r}, \ T = T_w(\mathrm{z,t})$$

$$\mathrm{at\,r = at} \ \ \mathrm{w} \to 0, \ \mathrm{T} \to T_\infty \ \ r \to \infty, \tag{3.5}$$

where $N = N_1\sqrt{1-\beta t}$ and has dimension (velocity)$^{-1}$. For N =0, the no-slip condition can be obtained. $T_w(\mathrm{z,t})$ is defined as the fluid surface temperature

$$T_w\left(z,t\right) = T_\infty + \frac{bz}{a_0 v(1-\beta\,\mathrm{t})} \tag{3.6}$$

The equation can be decreased to ODEs using the following dimensional variables

$$\eta = \left(\frac{r}{a_0} \right)^2 \frac{1}{1-\beta t}, \quad u = -\frac{1}{a_0^{\ 2}} \frac{2v}{\sqrt{1-\beta t}} \frac{f(\eta)}{\sqrt{\eta}},$$

$$w = \frac{1}{a_0^{\ 2}} \frac{4vz}{1-\beta t} f'(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{3.7}$$

By using Eq.(3.7) and Eqs.(3.3)-(3.4) is satisfied automatically and becomes

$$\eta f''' + f'' + ff'' - f'^2 - S(\eta f'' + f') = 0 \ , \tag{3.8}$$

$$\eta\theta'' + \theta' + \Pr(f\theta' - \theta f') - S\Pr(\eta\theta' + \theta) = 0 \ , \tag{3.9}$$

Subject to B.C's

$$f(1) = \gamma, \quad f'(1) = \varepsilon + \lambda f'(1), \quad \theta(1) = 1 \ , \tag{3.10}$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0, \tag{3.11}$$

Where prime is representing differentiation w.r.t η , $\gamma = -\dfrac{a_0 U}{2v}$, $S = \dfrac{a_0^{\,2}\beta}{4v}$

$\lambda = 2N_1 \dfrac{\rho V}{a_0}$ and the $\Pr = \dfrac{v}{\alpha}$. From Eq.(3.2) pressure can be obtained as

$$\frac{p}{\rho} = const + v\left(u_r + \frac{u}{r}\right) - \frac{1}{2}u^2 + \int u_t dr \ , \tag{3.12}$$

The physical quantities of significance such as the Nu and C_f can be defined as

$$C_f = \frac{\tau_w}{\frac{\rho w_w^2}{2}} , \quad Nu = \frac{a(\mathrm{t})\,\mathrm{q}\,w}{k(T_w - T_\infty)} \tag{3.13}$$

The q_w and τ_w at wall are

$$\tau_w = \left(w_r\right)_{r=a} = \frac{1}{a_0^{\,3}} \frac{8v\mu z}{1-\beta t} f'(1),$$

$$q_w = -k(T_r)_{r=a} = -\frac{2kbz\theta'(1)}{\mathrm{a}^2{}_0\mathrm{v}(1-\beta\,\mathrm{t})^{\frac{3}{2}}} , \tag{3.14}$$

With the use of Eqs. Eq. (3.7) and Eq. (3.13). (3.12) we obtained

$$\frac{C_f z}{a} = f'(1), \quad Nu = -2\theta'(1) \tag{3.15}$$

3.3 Results and discussion

Nonlinear ODEs are solved numerically by using shooting technique and R.K method. For instance, the physical parameters experienced in the issue like speed and temperature distribution including C_f and thermal transfer rate are examined at the wall are displayed graphically .In the present study of the shrinking cylinder, we obtained dual solutions using various initial estimates of the missing value and in which all the profiles of temperature and velocity satisfy the boundary conditions for infinity related to shape and thickness of boundary layer with parameter values accordingly. Repeat to achieve the precision up to the iteration method. The comparison of current outcomes with current numerical outcomes is provided to demonstrate the precision and sustainability of the current technique and is obtained in good agreement.

Figure 3.2 shows the difference of $f''(1)$ against S.It is noticed that dual solutions exist for $f'(1)$ for some values of S such that for $S < S_c$. However, it is observed that first solution is more efficient than the second (researched by in the event of no slip condition in the impermeable stretching cylinder.Figure:.3.3 depicts $f'(1)$ variation against S for certain values of γ in case of $\lambda = 0.1$ It is also noted that the C_f magnitude increases with an increase in γ,while the magnitude of C_f decreases with an increase in S.

Figure 3.4 elucidates the difference in C_f $f'(1)$ against γ for certain values of λ and for $S = -1$ the dual solution exists. It is also discovered that the magnitude of the $f'(1)$ is improved by the λ. In Figure 3.5, we can see the impact of γ on the $f'(\eta)$ when $\lambda = 0.1$ are fixed. Furthermore, it can be seen that the $f'(\eta)$ increases with the rise in γ. This is due to the reality that the vorticity of the shrinking cylinder is not limited in physical fields unless the unconfined suction at the border is manipulated. For this reason, mass suction only happens when the flow is smooth and the fluid is denser on the surface.

Figure 3.6 illustrates the difference in $f'(\eta)$ for different values of S with the fixed γ and λ. It is also noticed that for negative values of S, two fluid velocity profiles exist. Furthermore, it is also seen that the $f'(\eta)$ for first solution shows positive gradient whereas for second solution the negative. Figure 3.7 displays the impact of the λ on the $f'(\eta)$ with fixed values of $S=-1$ and $\gamma=2$. From this Figure it is examined that $f'(\eta)$ profile within the boundary layer reduces as the λ.

Figure 3.8 describes the effect of Nu against S for some values of γ with fixed value of λ. It can be seen that the magnitude of Nu is reduces with an increase in S

for both solutions. On the other hand the magnitude of Nu is increases for first and decreases for second solution on increasing the value of γ.

Figure 3.9 depicts the impact of P_r on the Nu against S when γ =2 and λ =0.1are fixed. In this Figure it is observed that for fixed value of P_r, the magnitude of Nu reduced, $\theta'(1)$ is decreased on increasing the value of S whereas when P_r is increased by some values the magnitude of reduced Nu is increased for both solutions. Table 1 shows the unsteady flow due to stretching cylinder and comparison of present results and results of previous work. Similarly Table 2.1 expresses the comparison of Nu and C_f for different parameters.

Table 2.1 Results of $f''(1)$ and $f'(1)=-1$ for some values of S and γ where $*$ are the results for $\lambda=0$ by Zaimi et al. [16]

S	γ=0.1	γ=1.0	γ= 1.5	γ= 2
-4.0	3.84077 (3.84077)∗ -24.88224 (-24.88224)∗	4.78798 (4.78798)∗ -34.68662 (-34.68662)∗	5.30763 (5.30763)∗ -41.10070 (-41.10070)∗	5.82402 (5.82402)∗ -48.26178 (-48.26178)∗
-3.5	3.29909 (3.29909)∗ -17.47316 (-17.47316)∗	4.26102 (4.26102)∗ -25.35982 (-25.35982)∗	4.78575 (4.78575)∗ -30.59883 (-30.59883)∗	5.30593 (5.30593)∗ -36.50662 (-36.50662)∗
-3.0	2.73978 (2.73978)∗ -11.45527 (-11.45527)∗	3.72551 (3.72551)∗ -17.59911 (-17.59911)∗	4.25779 (4.25779)∗ -21.75543 (-21.75543)∗	4.78338 (4.78338)∗ -26.49973 (-26.49973)∗
-2.5	2.14665 (2.14665)∗ -6.71496 (-6.71496)∗	3.17614 (3.17614)∗ -11.30680 (-11.30680)∗	3.72055 (3.72055)∗ -14.47701 (-14.47701)∗	4.25433 (4.25433)∗ -18.15036 (-18.15036)∗
-2	1.46820	2.60122	3.16774	3.71509

	(1.46820)*	**(2.60122)***	**(3.16774)***	**(3.71509)***
	-3.10294	**-6.38034**	**-8.66897**	**-11.36813**
	(-3.10294)*	**(-6.38034)***	**(-8.66897)***	**(-11.36813)***

Table 2.2 Results of $f''(1)$ and Nu, $f'(1)=-1$ for $S,\ \lambda,\ \gamma$ and $\Pr$.

S	λ	Pr	f″(1)		θ′(1)	
			γ=0.5	γ=1.5	γ=0.5	γ=1.5
-3.5	0.	0.	2.17574 (-10.15767)	2.47322 (-14.22440)	-4.00123 (-2.94271)	-4.57733 (-3.19992)
-3.0			1.99396 (-7.16568)	2.33308 (-10.61181)	-3.58562 (-2.70312)	-4.15678 (-2.96765)
-2.5			1.779918 (-4.59890)	2.174206 (-7.43207)	-3.15434 (-2.44886)	-3.72632 (-2.72068)
-2.0			1.51574 (-2.46013)	1.99094 (-4.69920)	-2.69839 (-2.17697)	-3.27675 (-2.45435)
-3.0	0		3.18357 (-13.98628)	4.25779 (-21.75561)	-3.51936 (-2.84277)	-4.09036 (-3.19334)
	1.		0.55876 (-1.5767)	0.58014 (-2.3470)	-3.6552 (-2.6764)	-4.2119 (-2.8828)
	3		0.30415 (-0.81994)	0.31026 (-1.22715)	-3.66671 (-2.68234)	-4.22017 (-2.88100)
	0.	1.	1.25774 (-4.00685)	1.37766 (-5.91160)	-4.741771 (-3.61365)	-5.57990 (-4.08435)
		5.	1.25774	1.377665	-18.93601	-23.67497

			(-4.00685)	(-5.911602)	(-17.41302)	(-21.95005)
		7.	1.25774 (-4.00685)	1.37766 (-5.91160)	-25.95441 (-24.42747)	-32.68147 (-30.98041)
		10	1.25774 (-4.00685)	1.37766 (-5.91160)	-36.46867 (-34.94367)	-46.18604 (-44.50639)
		20	1.25774 (-4.00685)	1.37766 (-5.91160)	-71.48551 (-69.96797)	-91.19106 (-89.53913)

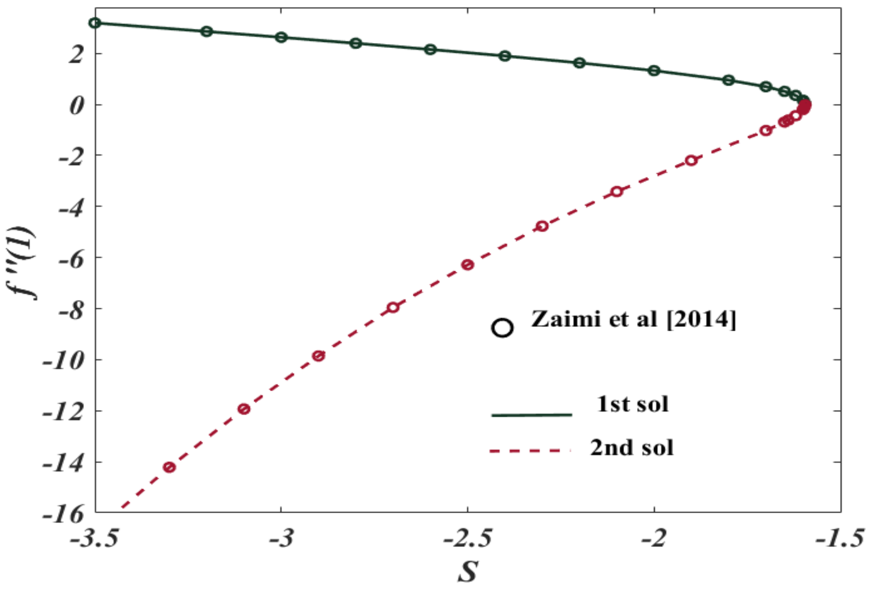

Figure 3.2 C_f with S for $\lambda = 0$

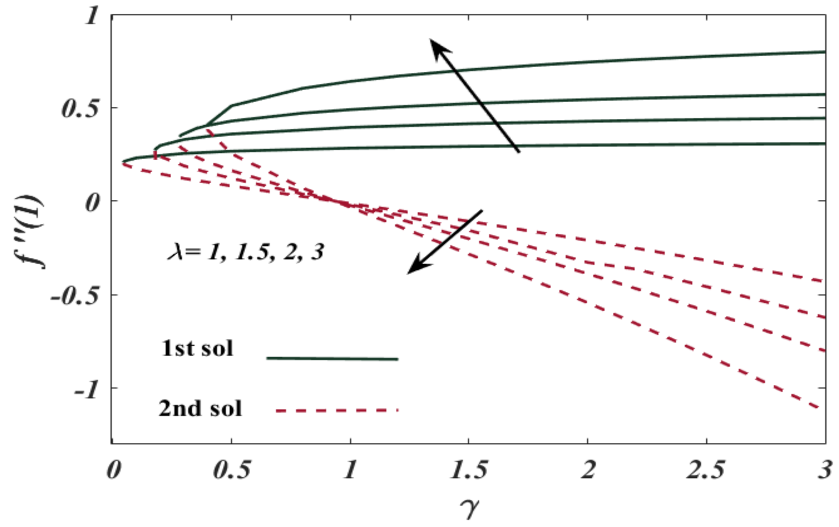

Figure 3.3 C_f **corresponding to** S **for** $\lambda = 0.1$

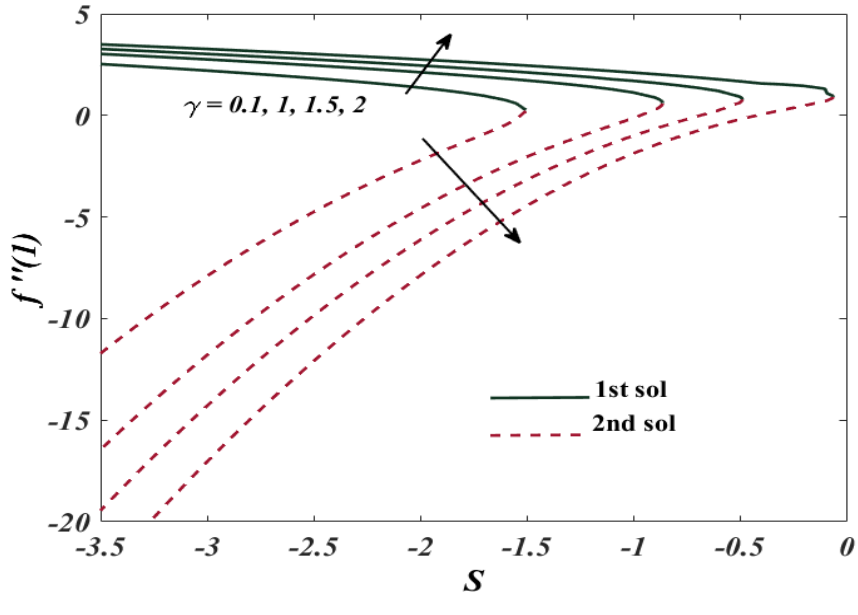

Figure 3.4 C_f **with** γ **for** $S = 0.1$

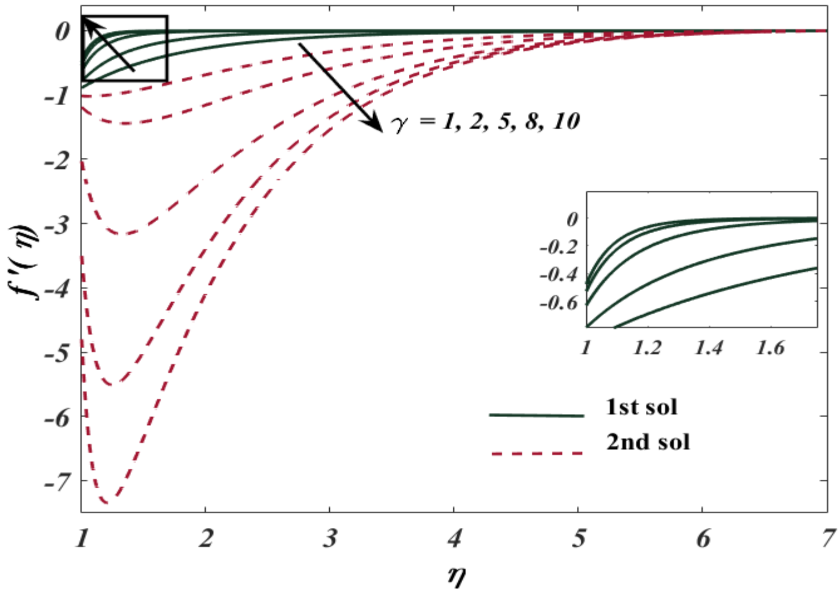

Figure 3.5 γ **on** $f'(\eta)$ **for** $S=-1$ **and** $\lambda=0.1$

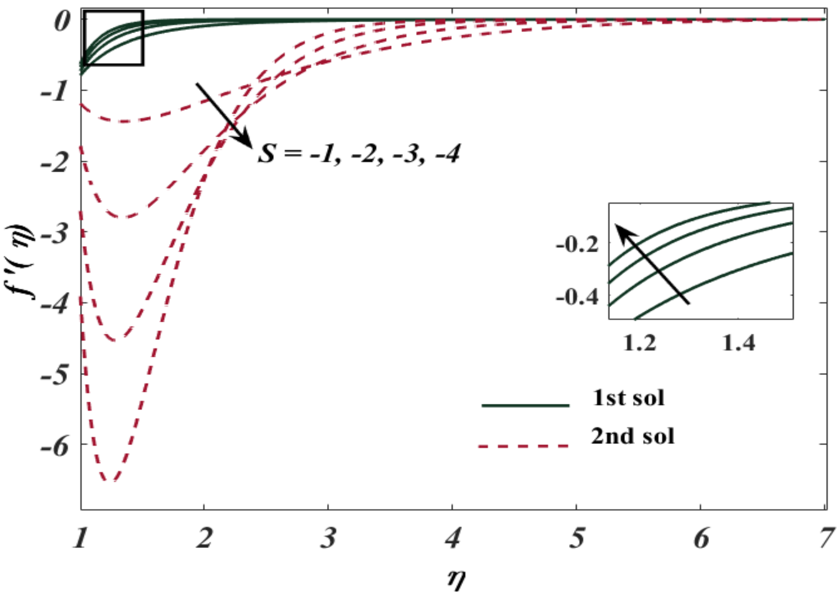

Figure 3.6 S **on** $f'(\eta)$ **for** γ **and** $\lambda=0.1$

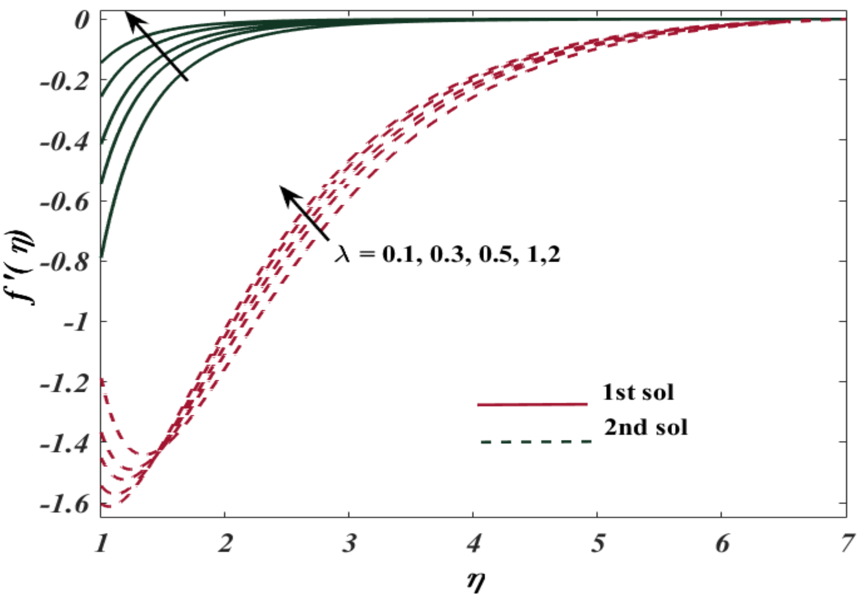

Figure 3.7 λ **on** $f'(\eta)$ **for** $\gamma = 2$ **and** $S = -1$

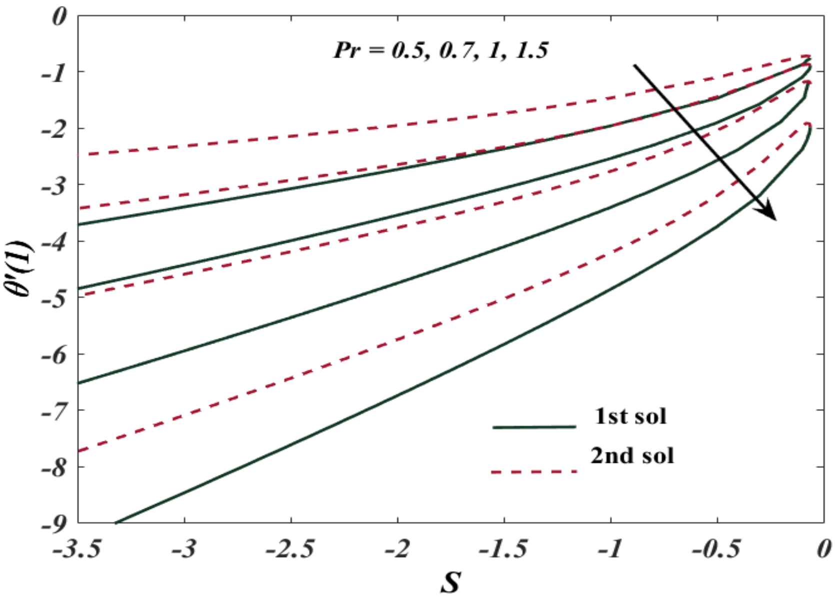

Figure 3.8 Nu **on** $\theta'(\eta)$ **with** S **for** $\Pr = 0.7$ **and** $\lambda = 0.1$

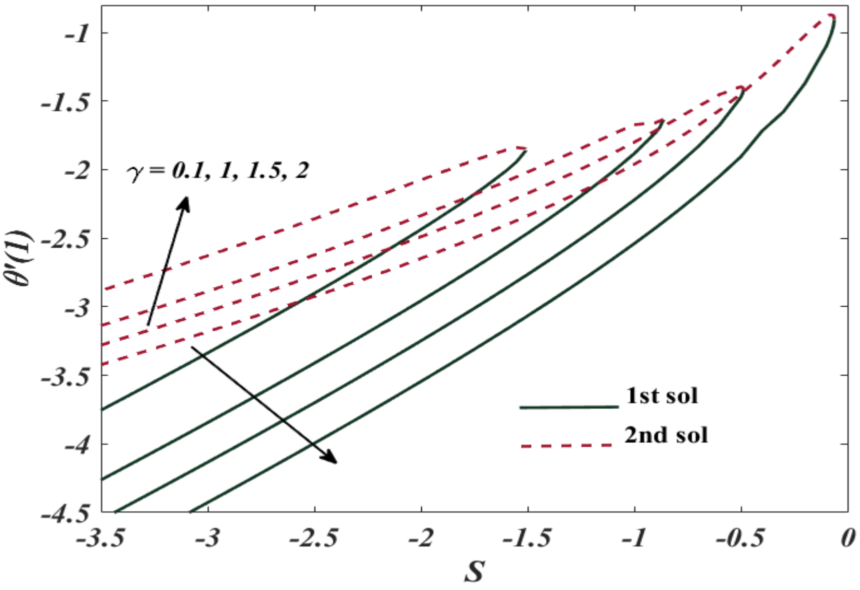

Figure 3.9 Nu **on** $\theta'(1)$ **for** S **when** $\lambda = 0.1$ **and** $\gamma = 2$

Chapter 4

Study of nanofluid flow with unequal diffusivities of H-H reactions past a stretching/shrinking cylinder

The main objective is to explore the results of unequal diffusivities of homogeneous-heterogeneous (H-H) responses on the nanofluid flowing over a shrinking cylinder. The cylinder is exposed to a perpendicular driven magnetic field. The generalized slip effects will also be studied. We also analyzed our results including the existence of MHD. With the assistance of similarity transformations, the ensuring PDEs are modified in dimensionless ODEs equation and by the use of shooting technique the resulting numerical solutions will be obtained. Physical analysis of appropriate parameters such as , ϕ, M, α, β, $\Pr$, Sc, K_p, γ, k_f in the distribution of temperature and velocity are shown in graphs and discussed in detail.

4.1 Problem Formulation

The geometry of the issue under consideration shown in Figure 4.1 consist of permeable stretching/ shrinking cylinder having unsteady, two dimensional and incompressible viscous fluid flow with generalized slip velocity. The temperature at the cylinder surface is $T_w(z,t)$ and where $T_w > T_\infty$ corresponds to assisting flow. In the radial direction B_0 is applied. Further the effects of H-H reactions are also considered in the model in the following form

$$2B + A \rightarrow 3B, \qquad rate = k_c a b^2$$

While isothermal reaction in the first order is transmitted in the way of the catalyst surface $A \rightarrow B$, $\text{rate} = k_p a$

,

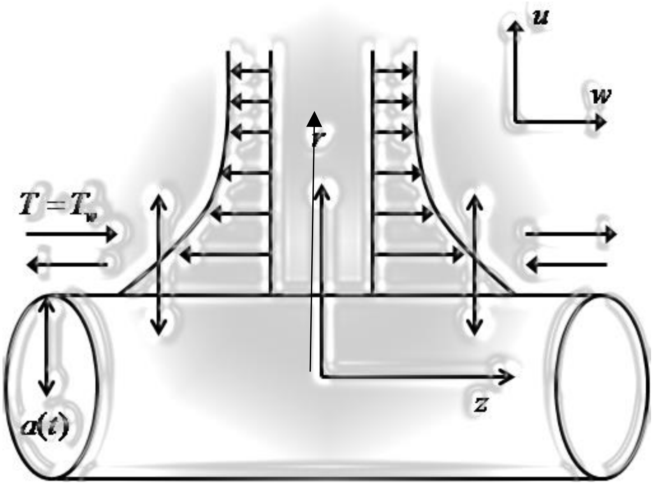

Figure 4.1 Configuration physique of the system

The basic equations under these assumptions are given by

$$\frac{1}{r}(ru)_r + w_z = 0 \tag{4.1}$$

$$u_t + uu_r + wu_z = -\frac{1}{\rho_{nf}} p_r + v\left(u_{rr} + \frac{1}{r}u_r + u_{zz} - \frac{u}{r^2}\right), \tag{4.2}$$

$$w_t + uw_r + ww_z = -\frac{1}{\rho_{nf}} p_z + v\left(w_{rr} + \frac{1}{r}w_r + w_{zz}\right) - wB_1^{\,2}\frac{\sigma_{nf}}{\rho_{nf}}, \tag{4.3}$$

$$(\rho c_p)_{nf}\left(T_t + uT_r + wT_z\right) = k_{nf}\left(T_{rr} + \frac{1}{r}T_r + T_{zz}\right), \tag{4.4}$$

$$a_t + ua_r + wa_z = D_A\left(a_{rr} + \frac{1}{r}a_r\right) - k_c ab^2, \tag{4.5}$$

$$b_t + ub_r + wb_z = D_B\left(b_{rr} + \frac{1}{r}a_r\right) + k_c ab^2. \tag{4.6}$$

Here the parameters are defined as

$$\sigma_{nf} = \sigma_f\left(1 + \frac{3(\sigma-1)\phi}{(\sigma+2)(\sigma-1)}\right)\ \rho_{nf} = \varphi\rho_s + (1-\varphi)\rho_f\ ,\ \frac{\kappa_{nf}}{\kappa_f} = \frac{(\kappa_s+\kappa_f) - 2\varphi\left(\kappa_f - \kappa_s\right)}{(\kappa_s+\kappa_f) + \varphi\left(\kappa_f - \kappa_s\right)},$$

$$\left(\rho C_p\right)_{nf} = (1-\varphi)\left(\rho C_p\right)_f + \varphi\left(\rho C_p\right)_s$$

By supposing generalized slip velocity condition is given by

$$u_t(x,t) = \alpha * (1 - \beta * \tau_w)^{-\frac{1}{2}}$$

Thus, we assume that the BCs of equations (4.1)-(4.6)are:

$$u=\frac{U}{\sqrt{1-\beta t}}, \quad w=\varepsilon\frac{1}{a_0{}^2}\frac{4vz}{1-\beta t}+\alpha^*\left(1-\beta^* w_r\right)^{-\frac{1}{2}}w_r$$

$$T=T_w(\mathrm{z},\mathrm{t}) \quad \text{at} \quad \mathrm{r}=\mathrm{a}(\mathrm{t}), \quad D_A a_r=k_p a, \quad D_B b_r=-k_p a$$

$$\mathrm{w}\to 0, \quad \mathrm{T}\to T_\infty, \quad a\to a_0, \quad b\to 0, \quad r\to\infty,$$

(4.7)

Using the following dimensional variables and parameters, the equation can be conver ted to ordinary differential equations

$$\eta=\left(\frac{r}{a_0}\right)^2\frac{1}{1-\beta t}, \quad u=-\frac{1}{a_0{}^2}\frac{2v}{\sqrt{1-\beta t}}\frac{f(\eta)}{\sqrt{\eta}}, \qquad w=\frac{1}{a_0{}^2}\frac{4vz}{1-\beta t}f'(\eta),$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \quad a=g(\eta)a_0, \quad b=h(\eta)\mathrm{b}_0$$

(4.8)

With the help of 4.7 and 4.8 the above equations 4.1 to 4.6 are transformed into

$$\eta f'''+ff''+f''-f'^2-S(\eta f''+f')+M^2 f'\frac{A_2}{A_1}=0,$$

(4.9)

$$\eta\theta''+\theta'+\frac{A_3}{A_4}\Pr(f\theta'-\theta f')-S\Pr(\eta\theta'+\theta)=0$$

,

(4.10)

$$\frac{1}{Sc}[g'+\eta g'']-\mathrm{S}\eta g'+fg'-Kgh^2,$$

(4.11) $$\frac{\delta}{Sc}[\mathrm{h}'+\eta h'']-\mathrm{S}\eta h'+fh'+Kgh^2.$$

(4.12)

Here the constants are defined as

$$A_1=\varphi\frac{\rho_s}{\rho_f}+(1-\varphi), \ \mathrm{A}_2=\frac{1+3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}, \ \mathrm{A}_3=\varphi\left(\rho C_p\right)_s\left(\frac{1}{\left(\rho C_p\right)_f}\right)+(1-\varphi),$$

$$A_4=\kappa_{nf}\left(\frac{1}{\kappa_f}\right)=\frac{(\kappa_s+2\kappa_f)-2\varphi\left(\kappa_f-\kappa_s\right)}{(\kappa_s+2\kappa_f)+\varphi\left(\kappa_f-\kappa_s\right)}$$

Table 4.1 Thermo physical quantities of copper particles and water

Liquid/ NPs	ρ (kg/m^3)	C_p (J/kg K)	κ (W/m K)	$\sigma(S/m)$
Water	997.1	4179	0.613	5.5×10^{-6}
Copper	8933	385	400	5.96×10^{7}

With BCs

$$f(1)=\gamma, \quad f'(1)=\varepsilon+\lambda\alpha(\mathrm{t})\left[(1-\beta t * f''(1))^{\frac{-1}{2}}\right] f''(1),$$

$$\theta(1)=1,$$

(4.13)

$$g'(\eta)=K_p g(1), \quad \mathrm{h}'(\eta)=-K_p g(1), \quad f'(\infty)=0,$$

$$\theta(\infty)=0, \quad g(\infty)=1, \quad \mathrm{h}(\infty)=0$$

(4.14)

Where prime is representing differentiation with respect to η, $\gamma=-\dfrac{a_0 U}{2v}$,

$\mathrm{S}=\dfrac{a_0{}^2\beta}{4v}$, $\lambda=2N_1\dfrac{\rho V}{a_0}$ $\mathrm{Pr}=\dfrac{v}{\alpha}$. Here $K=\dfrac{k_c A_1{}^2(1-\beta t)a_0{}^2}{4v}$ is the parameter of homogeneous reaction signifying the strength of homogeneous reaction, $K_p=\dfrac{k_p a_0{}^2(1-\beta t)2\mathrm{r}}{D_A}$ measures the strength of heterogeneous reaction , $Sc=\dfrac{v}{D_A}$,

$$\delta=\frac{D_B}{D_A}$$

The quantities $\alpha(\mathrm{t})$ and $\beta(\mathrm{x},\mathrm{t})$ must be constants and not functions of the variable x and
t as in equation(4.13).This condition can be fulfilled if $\alpha(\mathrm{t})$ and $\beta(\mathrm{x},\mathrm{t})$ are considered as:

$$\alpha(\mathrm{t})=\alpha *(t), \qquad \beta(\mathrm{x},\mathrm{t})=\left(\frac{8vz}{a_o{}^3(1-\beta t)^{\frac{3}{2}}}\right)\beta *(x,t) \tag{4.15}$$

Furthermore, the chemical species of diffusion coefficients A and B are assumed not t o

Be of a similar size. This leads to more assumptions that the diffusion coefficients D_A and D_B are unequal, i.e

$$g(\eta)+h(\eta)\neq 1$$

4.2 Results and discussion

The above equations are analyzed with BC and results of numerical solution for temperature and velocity are discussed and are presented via graphs. The main objective is to observe the variation in various parameters i.e. ϕ, M, $\alpha, \beta, \Pr, \mathrm{Sc}$, K_p, γ on $f'(\eta)$, $\theta(\eta)$, $g(\eta)$ and $h(\eta)$.

Figure 4.2 shows the variation of ϕ on $f'(\eta)$, it can be clearly seen that for first solution it increases whereas for second solution it decreases. In Figure 4.3 we can see the effect of $f'(\eta)$ for several values of γ. Further it can be clearly seen that $f'(\eta)$ increases with increase in γ, near the surface of shrinking cylinder for first solution and decreases for second solution. This is the only reason that the mass suction occurs only when the flow is smooth. . Figure 4.4 displays the variation of β on $f'(\eta)$ can be clearly seen that for first solution it increases whereas for second solution it decreases. Figure 4.5 describes the effects of $f'(\eta)$ for different values of M in case of shrinking cylinder. It can be seen that with an increase in M is to increase in the velocity profile for first solution whereas an opposite behavior is noted for second solution.

Figure 4.6 we can easily see the impact of $\theta(\eta)$ on β for first and second solution. It can be observed that the temperature increases with an increase in the slip parameter. Figure 4.7 elucidates the effect of $\theta(\eta)$ on α, the boundary layer thickness for first solution is more than second and increase in α leads to increase in $\theta(\eta)$ .The effects of M on $\theta(\eta)$ is presented in Figure 4.8. It is noted that on increasing the values of M the $\theta(\eta)$ also increases for both first and second solution.

Figure 4.9 elucidates the effect of $\Pr$ on $\theta(\eta)$,it can be seen that on increasing values of $\Pr$ the $\theta(\eta)$ also decreases for both first and second solution in case of shrinking cylinder Figure 4.10 depict the $g(\eta)$, $h(\eta)$ for different values of kp .It can be seen that by increasing values of kp the $f'(\eta)$ decreases for both first and second solution. Figure 4.11 describes the variation of Sc on $g(\eta)$, $h(\eta)$. It shows that by increasing the values of Sc the concentration profile also increases for both first and second solution. As the diffusion coefficient species reduces, it is discovered that the reactant concentration increases at a faster rate. Figure 4.12 elucidates the variation of ϕ on $g(\eta)$, $h(\eta)$. It can be seen that by increasing value of ϕ both first and second solution increases

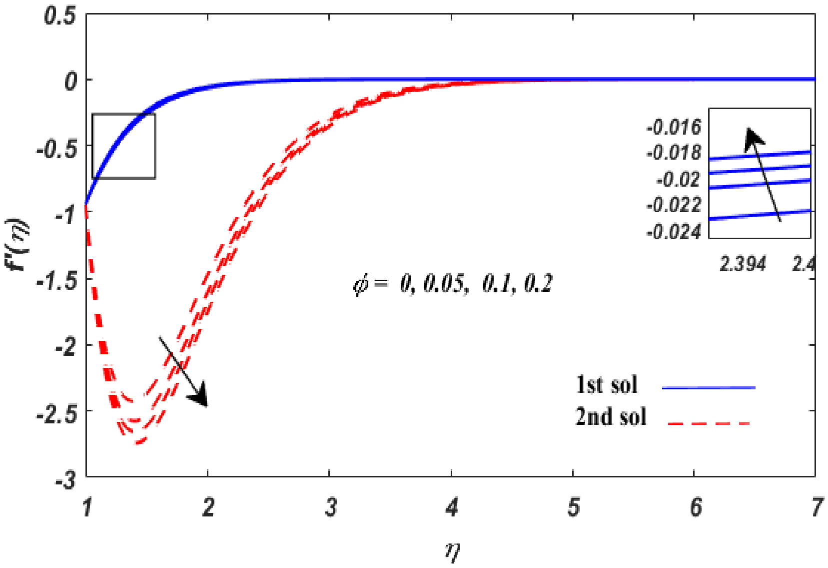

Figure 4.2 $f'(\eta)$ for ϕ

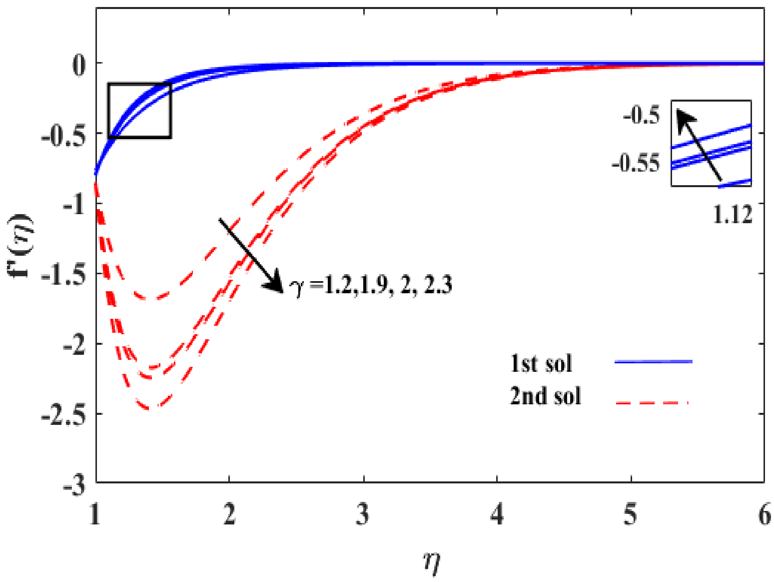

Figure 4.3 $f'(\eta)$ for γ

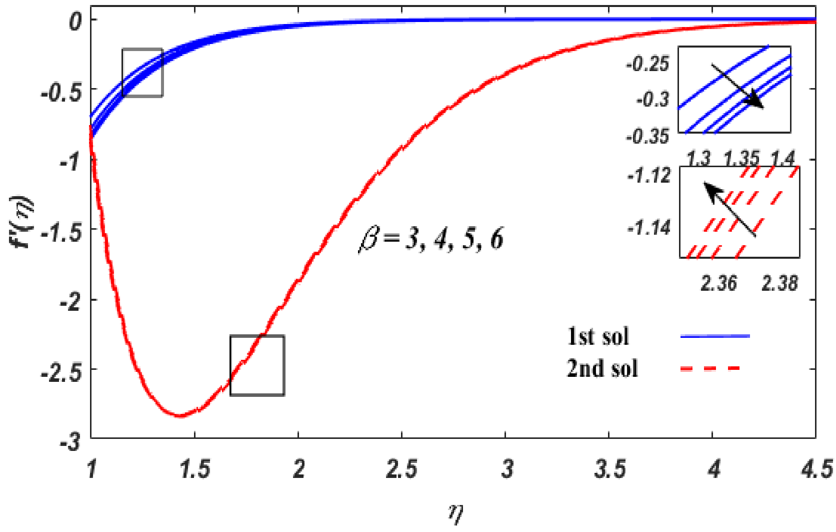

Figure 4.4 f $'(\eta)$ for β

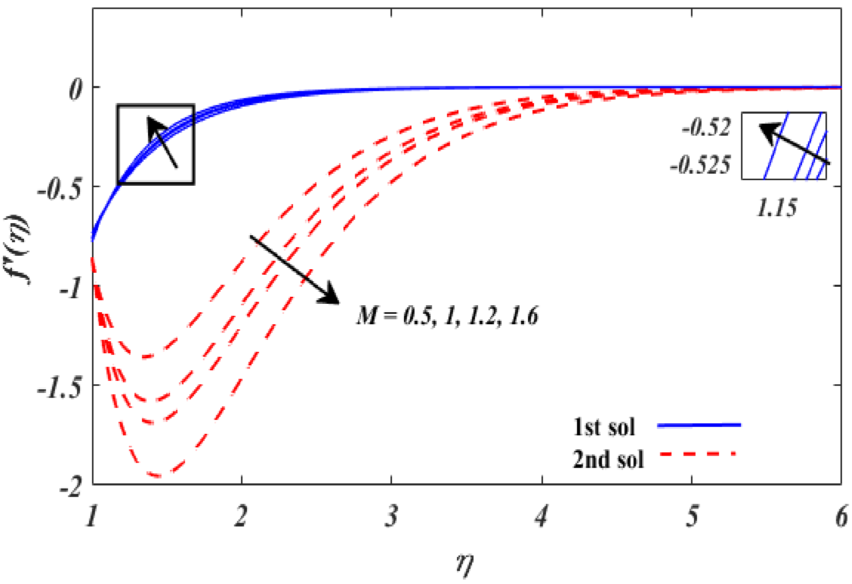

Figure 4.5 f $'(\eta)$ for M

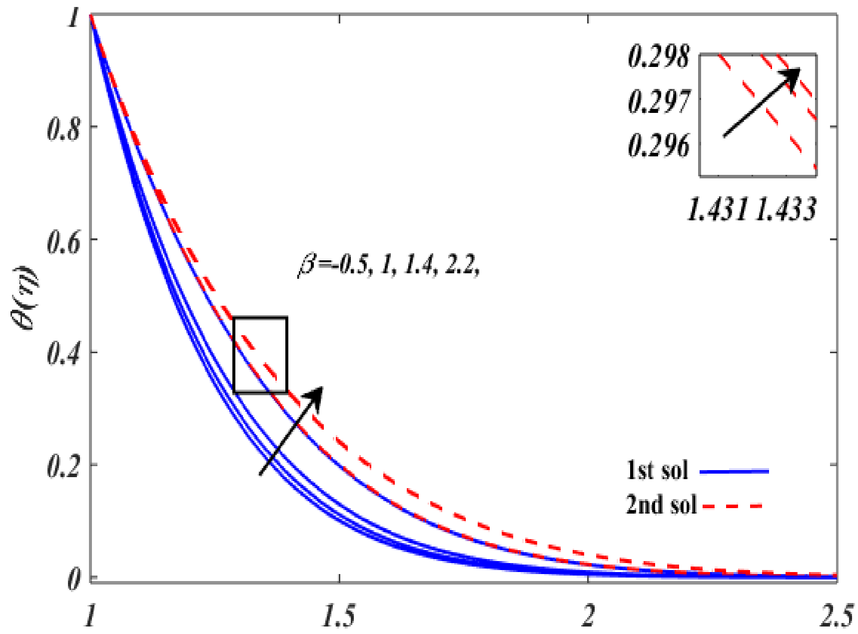

Figure 4.6 $\theta(\eta)$ *on* β

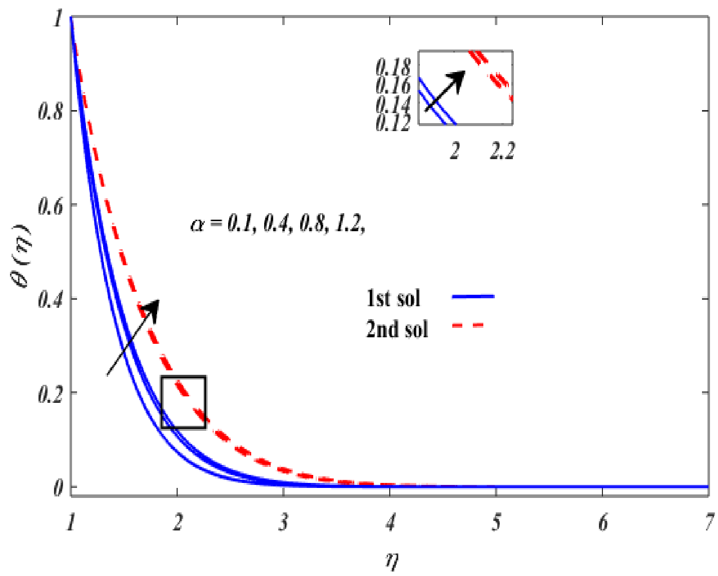

Figure 4.7 $\theta(\eta)$ on α

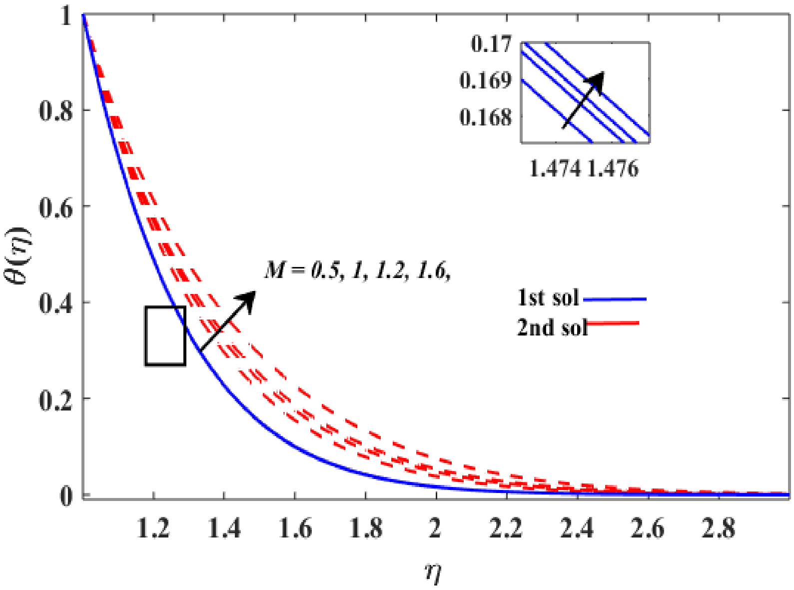

Figure 4.8 $\theta(\eta)$ **for** M

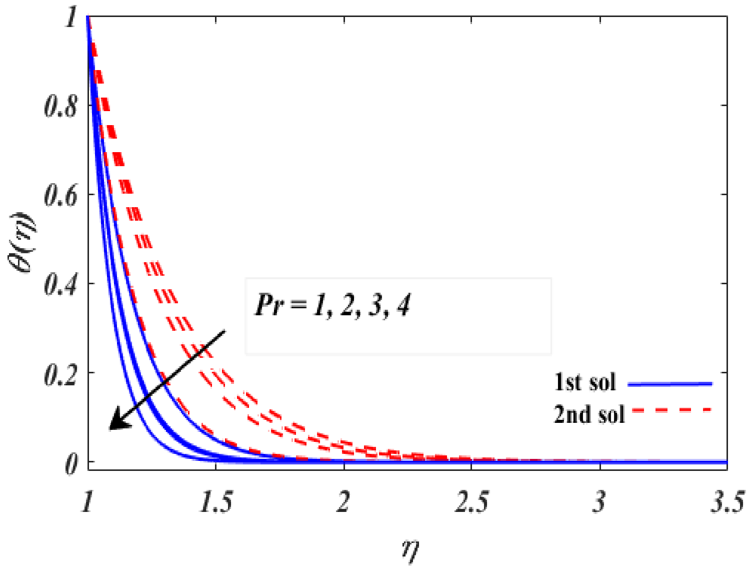

Figure 4.9 $\theta(\eta)$ **for** $\Pr$

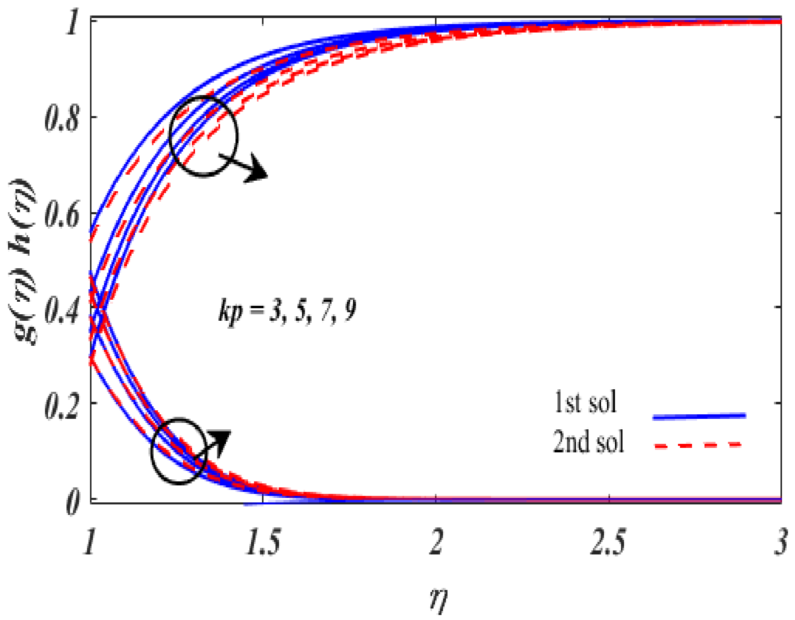

Figure 4.10 $g(\eta)\,\mathrm{h}(\eta)$ for kp

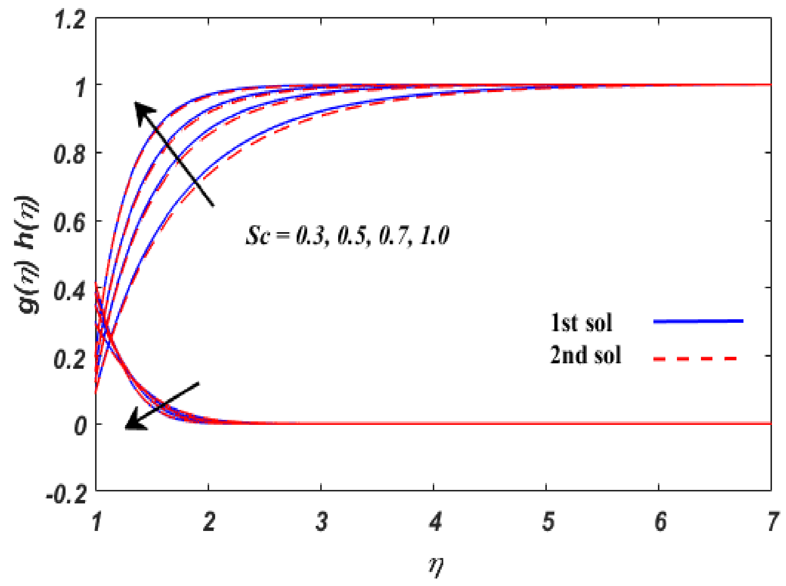

Figure 4.11 $g(\eta)\,\mathrm{h}(\eta)$ for Sc

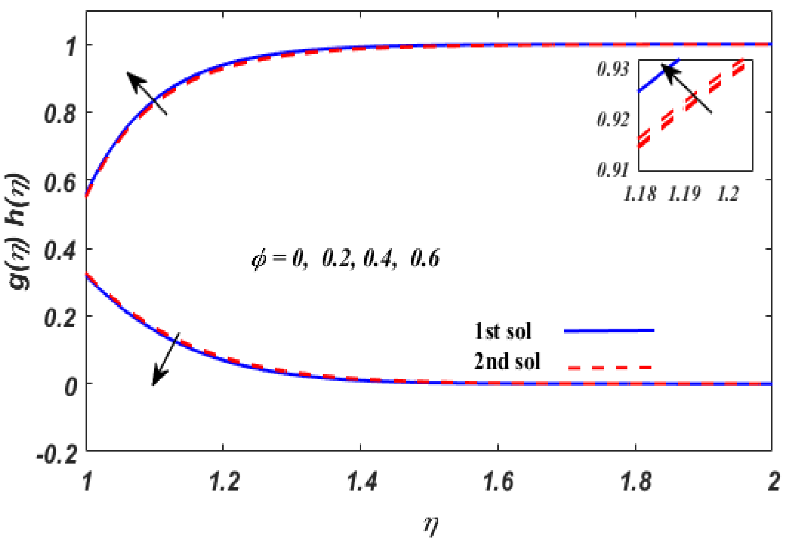

Figure 4.12 $g(\eta)\,\mathrm{h}(\eta)$ for ϕ

Chapter 5

Conclusion

In this research we studied nanofluid flow and heat transfer past a shrinking /stretching cylinder. We have mainly focused on slip, heat absorption/generation, suction, thermal and velocity while keeping remaining properties as constant. Similarity transformation helps to change the nonlinear PDEs into nonlinear ODEs. Resulting equations are solved by techniques. The impact of different governing parameters such as temperature, velocity and Nusselt number are investigated. The similarity solutions for the heat transfer assessment of unsteady viscous fluid are explored in the third section past a permeable shrinking cylinder with partial slip. The transformed ODEs were numerically solved by shooting method for multiple valu es of relevant parameters such as velocity flow, temperature field C_f & Nu .The findings are discussed and displayed in a graphical and tabular form. The following significant observations as under:

- Dual solutions are provided in this study in the event of shrinking cylinder.
- $f'(\eta)$ decreases in γ and $\lambda = 0.1$, while increases for greater values of S .
- γ and λ increases with the increase in C_f while for greater values of S it decreases.
- $\theta(\eta)$ reduces with increase in γ and $\Pr$, where as it increases for larger values of S .
- Nu decreases as S increases where as it increases with an increment in $\Pr$ and γ .

In chapter four our work has been extended, the unsteady viscous fluid past through stretching cylinder is discussed. The generalized slip effect in channel is also taken into consideration. The findings are also evaluated in the presence of MHD. The copper particles are added to the based fluid (water).

To demonstrate the impacts of the parameters $\phi, M, \alpha, \beta, \Pr, Sc$, K_p, γ on $f'(\eta)$, $\theta(\eta)$, the findings are provided through graphs. The conclusions of the study can be drawn as follows:

- $f'(\eta)$ are increased for greater values of φ for first solution and decreases for second solution.
- $f'(\eta)$ reduces for greater values of M.
- For larger values α, $f'(\eta)$ increases.
- $f'(\eta)$ reduces by increase in coefficients of heat absorption while reverse behavior is seen on increment in coefficients of heat generation.
- $\theta(\eta)$ increases on increment in M, while it decreases on increasing $\Pr$.
- For both first and second solution boundary layer thickness also increases for higher values of the H-H parameters.

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